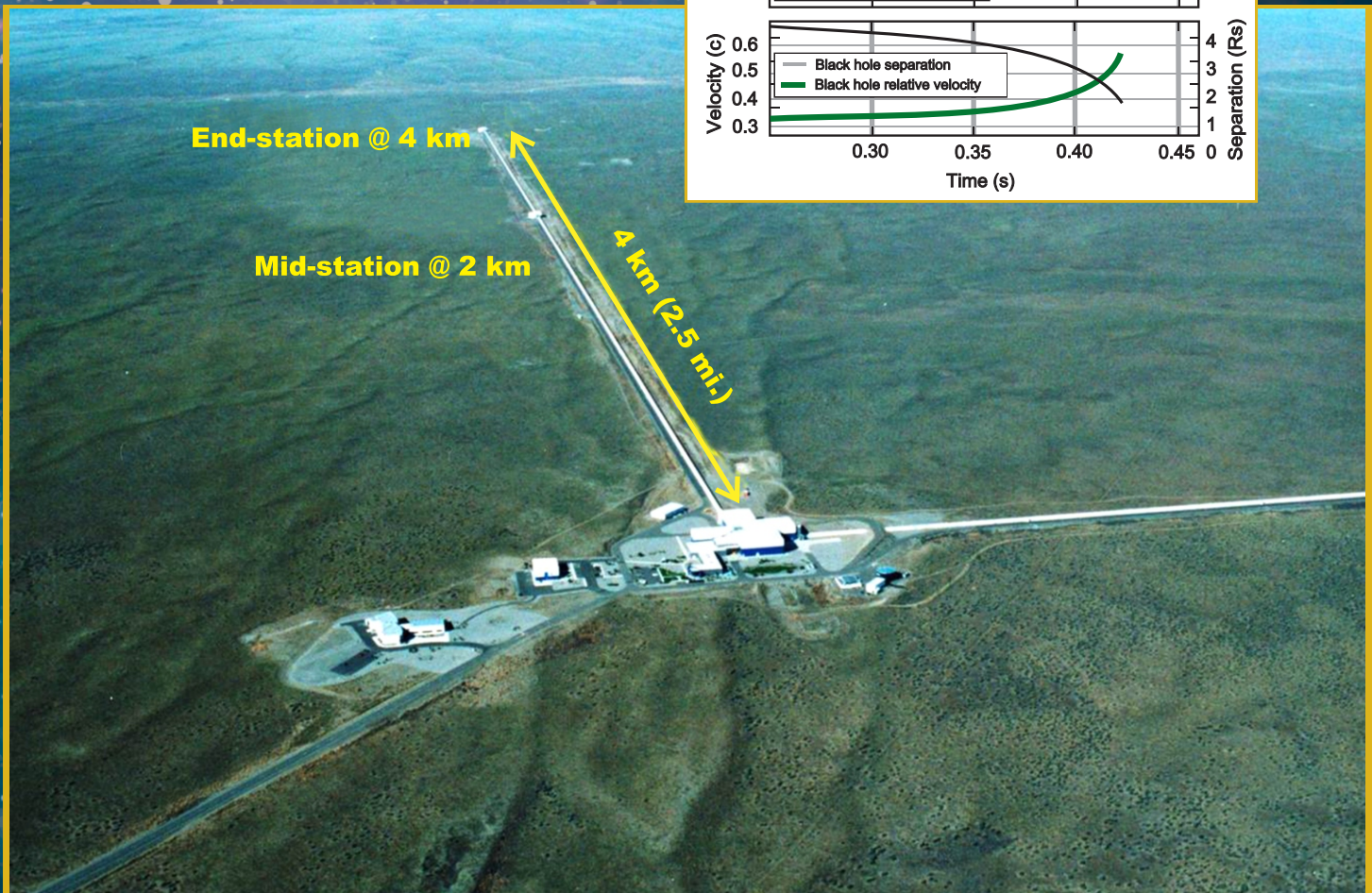
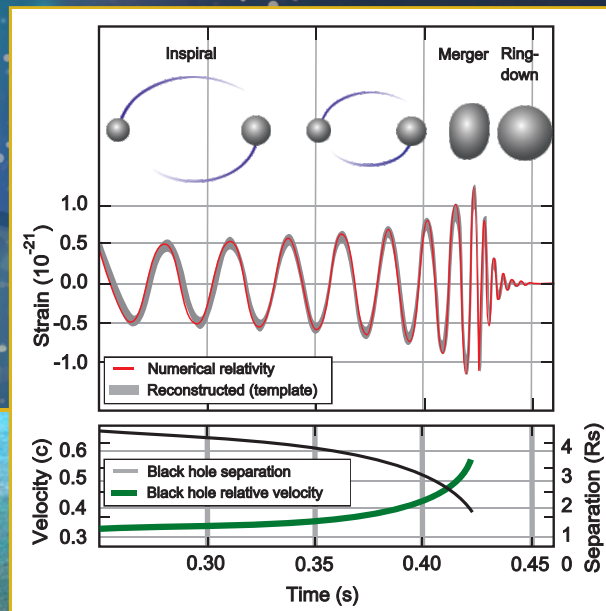




PHYSICS

Standard XI



The Constitution of India

Chapter IV A

Fundamental Duties

ARTICLE 51A

Fundamental Duties- It shall be the duty of every citizen of India—

- (a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;
- (b) to cherish and follow the noble ideals which inspired our national struggle for freedom;
- (c) to uphold and protect the sovereignty, unity and integrity of India;
- (d) to defend the country and render national service when called upon to do so;
- (e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities, to renounce practices derogatory to the dignity of women;
- (f) to value and preserve the rich heritage of our composite culture;
- (g) to protect and improve the natural environment including forests, lakes, rivers and wild life and to have compassion for living creatures;
- (h) to develop the scientific temper, humanism and the spirit of inquiry and reform;
- (i) to safeguard public property and to abjure violence;
- (j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;
- (k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

The Coordination Committee formed by GR No. Abhyas - 2116/(Pra.Kra.43/16) SD - 4 Dated 25.4.2016 has given approval to prescribe this textbook in its meeting held on 20.06.2019 and it has been decided to implement it from academic year 2019-20.

PHYSICS

Standard XI



T8P5A8

Download DIKSHA App on your smartphone. If you scan the Q.R.Code on this page of your textbook, you will be able to access full text. If you scan the Q.R.Code provided, you will be able to access audio-visual study material relevant to each lesson, provided as teaching and learning aids.



2019

**Maharashtra State Bureau of Textbook Production and
Curriculum Research, Pune.**

First Edition : © Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune - 411 004.
2019

Reprint: 2022

The Maharashtra State Bureau of Textbook Production and Curriculum Research reserves all rights relating to the book. No part of this book should be reproduced without the written permission of the Director, Maharashtra State Bureau of Textbook Production and Curriculum Research, 'Balbharati', Senapati Bapat Marg, Pune 411004.

Subject Committee:

Dr. Chandrashekhar V. Murumkar, Chairman
Dr. Dilip Sadashiv Joag, Member
Dr. Pushpa Avinash Khare, Member
Dr. Rajendra Shankar Mahamuni, Member
Dr. Anjali Lalit Kshirsagar, Member
Dr. Rishi Baboo Sharma, Member
Shri. Rajiv Arun Patole, Member Secretary

Study group:

Dr. Umesh Anant Palnitkar
Dr. Vandana Laxmanrao Jadhav Patil
Dr. Neelam Sunil Shinde
Dr. Radhika Gautamkumar Deshmukh
Dr. Prabhakar Nagnath Kshirsagar
Dr. Bari Anil Ramdas
Dr. Sutar Milind Madhusudan
Dr. Bodade Archana Balasaheb
Dr. Chavan Jayashri Kalyanrao
Smt. Chokshi Falguni Manish
Shri. Ramesh Devidas Deshpande
Shri. Vinayak Shripad Katdare
Smt. Pratibha Pradeep Pandit
Shri. Dinesh Madhusudan Joshi
Shri. Kolase Prashant Panditrao
Shri. Brijesh Pandey
Shri. Ramchandra Sambhaji Shinde
Smt. Taksale Mugdha Milind

Illustration

Shri. Pradeep Ghodke
Shri. Shubham Chavan

Cover

Shri. Vivekanand S. Patil

Typesetting

DTP Section, Textbook Bureau,
Pune

Co-ordination :

Shri. Rajiv Arun Patole
Special Officer for Physics

Paper :

70 GSM Creamwove

Print Order :

N/PB/2022-23/75,000

Printer

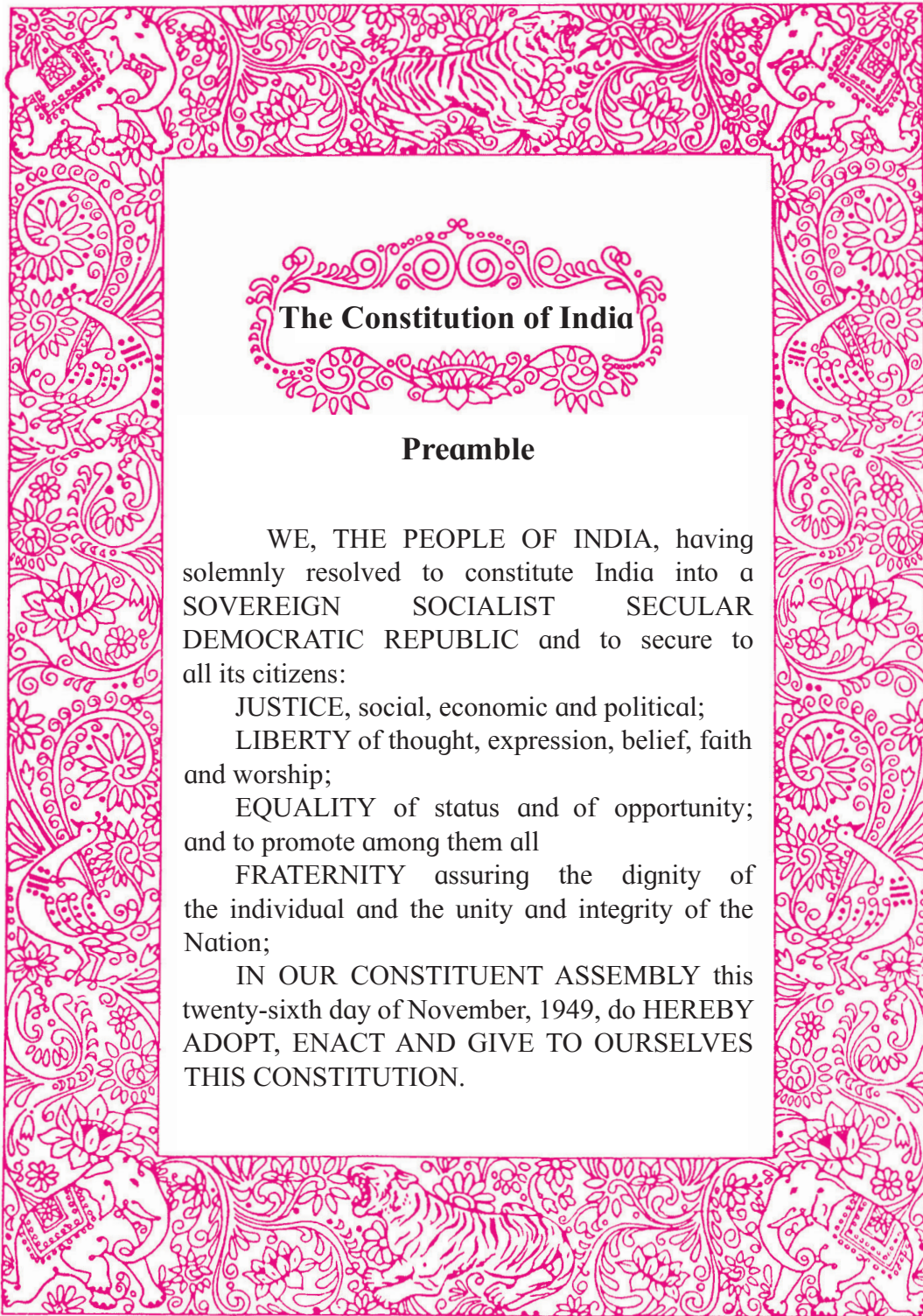
S. P. BINDING WORKS, PUNE

Production :

Shri Sachchitanand Aphale
Chief Production Officer
Shri Prashant Harne
Production Officer

Publisher :

Shri Vivek Uttam Gosavi
Controller
Maharashtra State Textbook
Bureau, Prabhadevi,
Mumbai - 400 025



The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens:

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā
uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians
are my brothers and sisters.

I love my country, and I am proud
of its rich and varied heritage. I shall
always strive to be worthy of it.

I shall give my parents, teachers
and all elders respect, and treat
everyone with courtesy.

To my country and my people,
I pledge my devotion. In their
well-being and prosperity alone lies
my happiness.

Preface

Dear Students,

It is a matter of pleasure and pride to place this exposition on basic physics in the hands of the young generation. This is not only textbook of physics for standard XI class, but embodies material which will be useful for self-study.

This textbook aims to create awareness about Physics. The National Curriculum Framework (NCF) was formulated in the year 2005, followed by the State Curriculum Framework (SCF) in 2010. Based on the given two frameworks, reconstruction of the curriculum and preparation of a revised syllabus has been undertaken which will be introduced from the academic year 2019-20. The textbook incorporating the revised syllabus has been prepared and designed by the Maharashtra State Bureau of Textbook Production and Curriculum Research, (Balharati), Pune.

The purpose of the book is to prepare a solid foundation for further studies in physics at the standard XII class. Proficiency in science in general and physics in particular is a basic requirement for the professional courses such as engineering and medicine etc., apart from the graduation courses in science itself. With this point of view, each chapter is prepared with elementary level and encompassing the secondary school level physics to the higher secondary level. Most of the topics are explained lucidly and in sufficient details, so that the students understand them well. A number of illustrative examples and figures are included to enlighten the student proficiency. With this background, the student is expected to solve the exercises given at the end of the chapters. For students who want more, Internet sites for many topics have been provided. They can enjoy further reading.

After all, physics is a conceptual subject. Knowledge about physical phenomena is gained as a natural consequence of observation, experience and revelation upon problem solving.

The book is written with this mind-set. The curriculum and syllabus conforms to the maxims of teaching such as moving from concrete to abstract, known to unknown and from part to the whole. For the first time, in this textbook of Physics, various activities have been introduced. These activities will not only help to develop understanding the content but also provide scope of the for gaining relevant and additional knowledge on your own efforts. A detailed information of all concepts is also given for a better understanding of the subject. QR Codes have been introduced for gaining additional information, abstracts of chapters and practice questions/ activities.

The efforts taken to prepare the textbook will not only enrich the learning experiences of the students, but also benefit other stakeholders such as teachers, parents as well as candidates aspiring for the competitive examinations.

We look forward to a positive response from the teachers and students.

Our best wishes to all!



(Dr. Sunil Magar)

Director

Maharashtra State Bureau of
Textbook Production and
Curriculum Research, Pune 4

Pune

Date : 20 June 2019

Bhartiya Saur : 30 Jyeshtha 1941

- For Teachers -

Dear Teachers,

We are happy to introduce the revised textbook of Physics for Std XI. This book is a sincere attempt to follow the maxims of teaching as well as develop a 'constructivist' approach to enhance the quality of learning. The demand for more activity based, experiential and innovative learning opportunities is the need of the hour. The present curriculum has been restructured so as to bridge the credibility gap that exists between what is taught and what students learn from direct experience in the outside world. Guidelines provided below will help to enrich the teaching-learning process and achieve the desired learning outcomes.

- ✓ To begin with, get familiar with the textbook yourself, and encourage the students to read each chapter carefully.
- ✓ The present book has been prepared for constructivist and activity-based teaching, including problem solving exercises.
- ✓ Use teaching aids as required for proper understanding of the subject.
- ✓ Do not finish the chapter in short.
- ✓ Follow the order of the chapters strictly as listed in the contents because the units are introduced in a graded manner to facilitate knowledge building.

- ✓ 'Error in measurements' is an important topic in physics. Please ask the students to use this in estimating errors in their measurements. This must become an integral part of laboratory practices.
 - ✓ Major concepts of physics have a scientific base. Encourage group work, learning through each other's help etc. Facilitate peer learning as much as possible by reorganizing the class structure frequently.
 - ✓ Do not use the boxes titled 'Do you know?' for evaluation. However, teachers must ensure that students read this extra information.
 - ✓ For evaluation, equal weightage should be assigned to all the topics. Use different combinations of questions. Stereotype questions should be avoided.
 - ✓ Use QR Code given in the textbook. Keep checking the QR Code for updated information. Certain important links, websites have been given for references. Also a list of reference books is given. Teachers as well as the students can use these references for extra reading and in-depth understanding of the subject.
- Best wishes for a wonderful teaching experience!

References:

1. Fundamentals of Physics - Halliday, Resnick, Walker; John Wiley (sixth ed.).
2. Sears and Zeemansky's University Physics - Young and Freedman, Pearson Education (12th ed.)
3. Physics for Scientists and Engineers - Lawrence S. Lerner; Jones and Bartlett Publishers, UK.

Front Page : Figure shows the LIGO laboratory in the United States of America and the inset shows the trace of gravity waves detected upon the merger of two black holes. In the background is the artist's impression of planets and galaxies.

Since ages, mankind is awed by the sheer scale of the universe and is trying to understand the laws governing the same. Today we observe the events in the universe with highly sophisticated instruments and laboratories such as the LIGO project seen on the cover. Picture Credit: Caltech/ MIT/ LIGO laboratory.

Figure Credit : B. P. Abott et al. Physical Review Letts 116, 061102, 2016

DISCLAIMER Note : All attempts have been made to contact copy right/s (©) but we have not heard from them. We will be pleased to acknowledge the copy right holder (s) in our next edition if we learn from them.

Competency Statements Standard XI

Area/ Unit/ Lesson	Competency Statements After studying the content in Textbook student ...
Units and Mathematical Tools	- Distinguish between fundamental and derived quantities. - Distinguish between different system of units and their use. - Identify methods to be used for measuring lengths and distances of varying magnitudes. - Check correctness of physical equations using dimensional analysis. - Establish the relation between related physical quantities using dimensional analysis. - Find conversion factors between the units of the same physical quantity in two different sets of units. - Identify different types of errors in measurement of physical quantities and estimate them. - Identify the order of magnitude of a given quantity and the significant figures in them. - Distinguish between scalar and vector quantities. - Perform addition, subtraction and multiplication (scalar and vector product) of vectors. - Determine the relative velocity between two objects. - Obtain derivatives and integrals of simple functions. - Obtain components of vectors. - Apply mathematical tools to analyze physics problems.
Motion and Gravitation	- Visualize motions in daily life in one, two and three dimensions. - Explain the necessity of Newton's first law of motion. - Categorize various forces of nature into four fundamental forces. - State various conservation principles and use these in daily life situations. - Derive expressions and evaluate work done by a constant force and variable force. - Organize/categorize the common principles between collisions and explosions. - Explain the necessity of defining impulse and apply it to collisions, etc. - Elaborate the limitations of Newton's laws of motion. - Elaborate different types of mechanical equilibria with suitable examples. - Apply the Kepler's laws of planetary motion to solar system. - Elaborate Newton's law of gravitation. - Calculate the values of acceleration due to gravity at any height above and depth below the earth's surface. - Distinguish between different orbits of earth's satellite. - Explain how escape velocity varies from planet. - Explain weightlessness in a satellite.
Properties of Matter	- Explain the difference between elasticity and plasticity - Identify elastic limit for a given material. - Differentiate between different types of elasticity modules. - Judge the suitability of materials for specific applications in daily life appliances. - Identify the role of force of friction in daily life. - Differentiate between good and bad conductors of heat. - Relate underlying physics for use of specific materials for use in thermometers for specific applications.
Sound and Optics	- Apply and relate various parameters related to wave motion. - Compare various types of waves with common features and distinguishing features. - Analytically relate the factors on which the speed of sound and speed of light depends. - Explain the essential factor to describe wave propagation and relate it with phase angle. - Apply the laws of reflection to light. - Mathematically describe the Doppler effect for sound waves. - Apply the laws of refraction to common phenomena in daily life like, a mirage or a rainbow. - Identify the defects in images obtained by mirrors and lenses, with their cause and ways of reducing or eliminating them. - Explain the construction and use of various optical instruments such as a microscope, a telescope, etc. - Relate dispersion of light with colour and apply it analytically with the help of prisms.

	- Describe dispersive power as a basic property of transparent materials and relate it with their refractive indices. - Analyze the time taken to receive an echo and calculate distance to the reflecting object. - Explain reverberation and acoustics.
Electricity and Magnetism	- Distinguish between conductors and insulators. - Apply coulomb's law and obtain the electric field due to a certain distribution of charges. - Define dipole, obtain the dipolar field. - Relate the drift of electrons in a conductor to resistivity - Calculate resistivity at various temperature. - Connect resistors in series and parallel combination. - Compare electric and magnetic fields. - Draw electric and magnetic lines of force. - Obtain magnetic parameters of the Earth. - Solve numerical and analytical problems.
Communication and Semiconductors	- Explain the properties of an electromagnetic wave. - Distinguish between mechanical waves and electromagnetic waves. - Identify different types of electromagnetic radiations from γ- rays to radio waves. - Distinguish between different modes of propagation of EM waves through earth's atmosphere. - Identify different elements of a communication system. - Explain different types of modulation and identify the types of modulation needed in given situation. - Distinguish between conductors, insulators and semiconductors based on band structure. - Differentiate between p type and n type semiconductors and their uses. - Explain working of forward and reverse biased junction. - Explain the working of semiconductor diode.

CONTENTS

Sr. No	Title	Page No
1	Units and Measurements	1-15
2	Mathematical Methods	16-29
3	Motion in a Plane	30-46
4	Laws of Motion	47-77
5	Gravitation	78-99
6	Mechanical Properties of Solids	100-113
7	Thermal Properties of Matter	114-141
8	Sound	142-158
9	Optics	159-187
10	Electrostatics	188-206
11	Electric Current Through Conductors	207-220
12	Magnetism	221-228
13	Electromagnetic Waves and Communication System	229-241
14	Semiconductors	242-256



Can you recall?

1. What is a unit?
2. Which units have you used in the laboratory for measuring
(i) length (ii) mass (iii) time (iv) temperature?
3. Which system of units have you used?

1.1 Introduction:

Physics is a quantitative science, where we measure various physical quantities during experiments. In our day to day life, we need to measure a number of quantities, e.g., size of objects, volume of liquids, amount of matter, weight of vegetables or fruits, body temperature, length of cloth, etc. A measurement always involves a comparison with a standard measuring unit which is internationally accepted. For example, for measuring the mass of a given fruit we need standard mass units of 1 kg, 500 g, etc. These standards are called units. The measured quantity is expressed in terms of a number followed by a corresponding unit, e.g., the length of a wire is written as 5 m where m (metre) is the unit and 5 is the value of the length in that unit. Different quantities are measured in different units, e.g. length in metre (m), time in seconds (s), mass in kilogram (kg), etc. The standard measure of any quantity is called the unit of that quantity.

1.2 System of Units:

In our earlier standards we have come across various systems of units namely

- (i) CGS: Centimetre Gram Second system
- (ii) MKS: Metre Kilogram Second system
- (iii) FPS: Foot Pound Second system.
- (iv) SI: System International

The first three systems namely CGS, MKS and FPS were used extensively till recently. In 1971, the 14th International general conference on weights and measures recommended the use of 'International system' of units. This international system of units is called the SI units. As the SI units use decimal system, conversion within the system is very simple and convenient.

1.2.1 Fundamental Quantities and Units:

The physical quantities which do not depend on any other physical quantities for their measurements are known as fundamental quantities. There are seven fundamental quantities: length, mass, time, temperature, electric current, luminous intensity and amount of substance.

Fundamental units: The units used to measure fundamental quantities are called fundamental units. The fundamental quantities, their units and symbols are shown in the Table 1.1.

Table 1.1: Fundamental Quantities with their SI Units and Symbols

Fundamental quantity	SI units	Symbol
1) Length	metre	m
2) Mass	kilogram	kg
3) Time	second	s
4) Temperature	kelvin	K
5) Electric current	ampere	A
6) Luminous Intensity	candela	cd
7) Amount of substance	mole	mol

1.2.2 Derived Quantities and Units:

In physics, we come across a large number of quantities like speed, momentum, resistance, conductivity, etc. which depend on some or all of the seven fundamental quantities and can be expressed in terms of these quantities. These are called derived quantities and their units, which can be expressed in terms of the fundamental units, are called derived units.

For example,

SI unit of velocity

$$= \frac{\text{Unit of displacement}}{\text{Unit of time}} = \frac{\text{m}}{\text{s}} = \text{m s}^{-1}$$

Unit of momentum = (Unit of mass) × (Unit of velocity)

$$= \text{kg m/s} = \text{kg m s}^{-1}$$

The above two units are derived units.

Supplementary units : Besides, the seven fundamental or basic units, there are two more units called supplementary units: (i) Plane angle $d\theta$ and (ii) Solid angle $d\Omega$

(i) **Plane angle ($d\theta$) :** This is the ratio of the length of an arc of a circle to the radius of the circle as shown in Fig. 1.1 (a). Thus $d\theta = ds/r$ is the angle subtended by the arc at the centre of the circle. It is measured in radian (rad). An angle θ in radian is denoted as θ^c .

(ii) **Solid angle ($d\Omega$) :** This is the 3-dimensional analogue of $d\theta$ and is defined as the area of a portion of surface of a sphere to the square of radius of the sphere. Thus $d\Omega = dA/r^2$ is the solid angle subtended by area ds at O as shown in Fig. 1.1 (b). It is measured in steradians (sr). A sphere of radius r has surface area $4\pi r^2$. Thus, the solid angle subtended by the entire sphere at its centre is $\Omega = 4\pi r^2/r^2 = 4\pi$ sr.

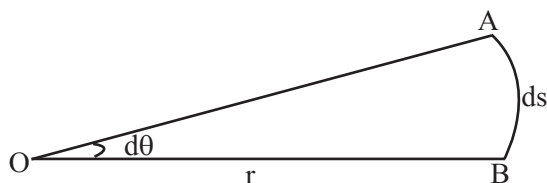


Fig 1.1 (a): Plane angle $d\theta$.

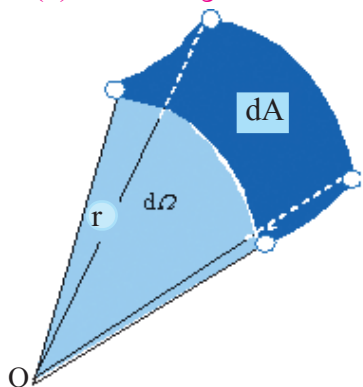


Fig 1.1 (b): Solid angle $d\Omega$.

Example 1.1: What is the solid angle subtended by the moon at any point of the Earth, given the diameter of the moon is 3474 km and its distance from the Earth 3.84×10^8 m.

Solution: Solid angle subtended by the moon at the Earth

$$\begin{aligned} &= \frac{\text{Area of the disc of the moon}}{(\text{moon - earth distance})^2} \\ &= \frac{\pi \times (1.737 \times 10^3)^2}{(3.84 \times 10^5)^2} \\ &= 6.425 \times 10^{-5} \text{ sr} \end{aligned}$$



Do you know ?

The relation between radian and degree is

$$\pi \text{ radians} = \pi^c = 180^\circ$$

$$\therefore 1 \text{ radian} = \frac{180}{\pi} = \frac{180}{3.1415} = 57.297^\circ$$

$$\text{Similarly } 1^\circ = \frac{\pi}{180} = \frac{3.1415}{180} = 1.745 \times 10^{-2} \text{ rad}$$

$$1^\circ = 60', 1' = 2.91 \times 10^{-4} \text{ rad}$$

$$\text{and } 1' = 60'', 1'' = 4.847 \times 10^{-6} \text{ rad}$$

1.2.3 Conventions for the use of SI Units:

- (1) Unit of every physical quantity should be represented by its symbol.
- (2) Full name of a unit always starts with smaller letter even if the name is after a person, e.g., 1 newton, 1 joule, etc. But symbol for unit named after a person should be in capital letter, e.g., N after scientist Newton, J after scientist Joule, etc.
- (3) Symbols for units do not take plural form for example, force of 20 N and not 20 newtons or not 20 Ns.
- (4) Symbols for units do not contain any full stops at the end of recommended letter, e.g., 25 kg and not 25 kg..
- (5) The units of physical quantities in numerator and denominator should be written as one ratio for example the SI unit of acceleration is m/s^2 or m s^{-2} but not m/s/s .
- (6) Use of combination of units and symbols for units is avoided when physical quantity is expressed by combination of two. e.g., The unit J/kg K is correct while joule/kg K is not correct.
- (7) A prefix symbol is used before the symbol of the unit.

Thus prefix symbol and units symbol constitute a new symbol for the unit which can be raised to a positive or negative power of 10.

1ms = 1 millisecond = 10^{-3} s

1 μ s = 1 microsecond = 10^{-6} s

1ns = 1 nanosecond = 10^{-9} s

Use of double prefixes is avoided when single prefix is available

10^{-6} s = 1 μ s and not 1mms.

10^{-9} s = 1ns and not 1m μ s

- (8) Space or hyphen must be introduced while indicating multiplication of two units e.g., m/s should be written as m s⁻¹ or m·s⁻¹ and not as ms⁻¹ (because ms will be read as millisecond).

1.3 Measurement of Length:

One fundamental quantity which we

have discussed earlier is length. To measure the length or distance the SI unit used is metre (m). In 1960, a standard for the metre based on the wavelength of orange-red light emitted by atoms of krypton was adopted. By 1983 a more precise measurement was developed. It says that a metre is the length of the path travelled by light in vacuum during a time interval of 1/299792458 second. This was possible as by that time the speed of light in vacuum could be measured precisely as $c = 299792458$ m/s.

Some typical distances/lengths are given in Table 1.2.

Table 1.2: Some Useful Distances

Measurement	Length in metre
Distance to Andromeda Galaxy (from Earth)	2×10^{22} m
Distance to nearest star (after Sun) Proxima Centuari (from Earth)	4×10^{16} m
Distance to Pluto (from Earth)	6×10^{12} m
Average Radius of Earth	6×10^6 m
Height of Mount Everest	9×10^3 m
Thickness of this paper	1×10^{-4} m
Length of a typical virus	1×10^{-8} m
Radius of hydrogen atom	5×10^{-11} m
Radius of proton	1×10^{-15} m

1.3.1 Measurements of Large Distance:

Parallax method

Large distance, such as the distance of a planet or a star from the Earth, cannot be measured directly with a metre scale, so a parallax method is used for it.

Let us do a simple experiment to understand what is parallax.

Hold your hand in front of you and look at it with your left eye closed and then with your right eye closed. You will find that your hand appears to move against the background. This effect is called parallax. Parallax is defined as the apparent change in position of an object due to a change in the position of the observer. By measuring the parallax angle (θ) and knowing the distance between the eyes E_1 and E_2 as shown in Fig. 1.2, we can determine the distance of the object from us, i.e., OP as E_1E_2/θ .

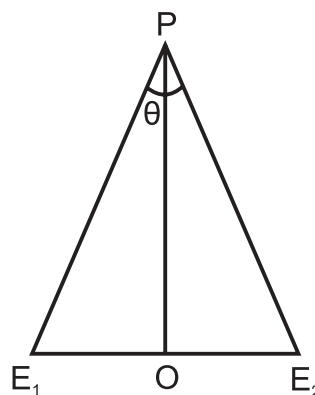


Fig.1.2: Parallax method for determining distance.

As the distances of planets from the Earth are very large, we can not use two eyes method as discussed here. In order to make simultaneous observations of an astronomical object, we select two distant points on the Earth.

Consider two positions A and B on the surface of Earth, separated by a straight line at

distance b as shown in Fig. 1.3. Two observers at these two points observe a distant planet S simultaneously. We measure the angle $\angle ASB$ between the two directions along which the planet is viewed at these two points. This angle, represented by symbol θ , is the parallax angle.

As the planet is far away, i.e., the distance of the planet from the Earth is very large in comparison to b , $b/D \ll 1$ and, therefore, θ is very small.

We can thus consider AB as the arc of length b of the circle and D its radius.

$AB = b$ and $AS = BS = D$ and $\theta \cong AB/D$, where θ is in radian

$$\therefore D = b / \theta \quad \text{--- (1.1)}$$

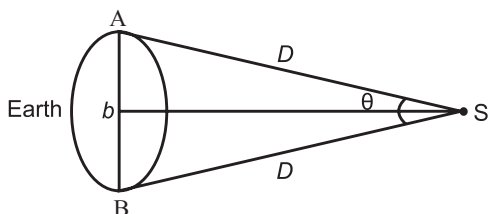


Fig.1.3: Measurement of distances of planets

1.3.2 Measurement of Distance to Stars:

Sun is the star which is closest to the Earth. The next closest star is at a distance of 4.29 light years. The parallax measured from two most distance points on the Earth for stars will be too small to be measured and for this purpose we measure the parallax between two farthest points (i.e. 2 AU apart, see box below) along the orbit of the Earth around the Sun (see figure in example 1.2 below).

1.3.3 Measurement of the Size of a Planet or a Star:

If d is the diameter of a planet, the angle subtended by it at any single point on the Earth is called angular diameter of the planet. Let α be the angle between the two directions when two diametrically opposite points of the planet are viewed through a telescope as shown in Fig. 1.4. As the distance D of the planet is large (assuming it has been already measured), we can calculate the diameter of the planet as

$$\alpha = \frac{d}{D}$$

$$\therefore d = \alpha D \quad \text{--- (1.2)}$$

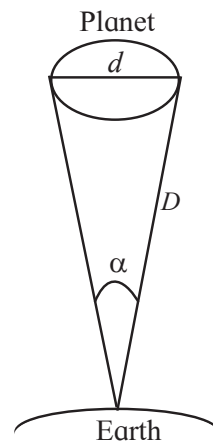


Fig. 1.4: Measurement of size of a planet

1.3.4 Measurement of Very Small Distances:

When we intend to measure the size of the atoms and molecules, the conventional apparatus like Vernier calliper or screw gauge will not be useful. Therefore, we use electron microscope or tunnelling electron microscope to measure the size of atoms.



Do you know ?

For measuring large distances, astronomers use the following units.

$$1 \text{ astronomical unit (AU)} = 1.496 \times 10^{11} \text{ m}$$

$$1 \text{ light year} = 9.467 \times 10^{15} \text{ m}$$

$$1 \text{ parsec (pc)} = 3.08 \times 10^{16} \text{ m} \cong 3.26 \text{ light years}$$

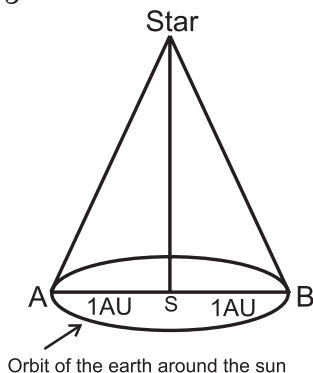
A light year is the distance travelled by light in one year. The astronomical unit (AU) is the mean distance between the centre of the Earth and the centre of the Sun.

A parsec (pc) is the distance from where 1AU subtends an angle of 1 second of arc.

$$r = \frac{1 \text{ AU}}{(1'')^c} = \frac{1.496 \times 10^{11}}{4.847 \times 10^{-6}} = 3.086 \times 10^{16} \text{ m}$$

Example 1.2: A star is 5.5 light years away from the Earth. How much parallax in arcsec will it subtend when viewed from two opposite

points along the orbit of the Earth?



Solution: Two opposite points A and B along the orbit of the Earth are 2 AU apart. The angle subtended by AB at the position of the star is = AB/distance of the star from the Earth

$$= \frac{2\text{AU}}{5.5 \text{ ly}} = \frac{2 \times 1.496 \times 10^{11} \text{ m}}{5.5 \times 9.46 \times 10^{15} \text{ m}} = 5.75 \times 10^{-6} \text{ rad}$$

$$= 5.75 \times 10^{-6} \times 57.297 \times 60 \times 60 \text{ arcsec}$$

$$= 1.186 \text{ arcsec}$$



Do you know ?

Small distances are measured in units of (i) fermi ($1\text{F} = 10^{-15} \text{ m}$) in SI system. Thus, 1F is one femtometre (fm) or (ii) Angstrom ($1 \text{ \AA} = 10^{-10} \text{ m}$).

For measuring sizes using a microscope we need to select the wavelength of light to be used in the microscope such that it is smaller than the size of the object to be measured. Thus visible light (wavelength from $4000 \text{ \AA}$ to $7000 \text{ \AA}$) can measure sizes down to about $4000 \text{ \AA}$. If we want to measure even smaller sizes we need to use even smaller wavelength and so the use of electron microscope is necessary. As you will study in the XIIth standard, each material particle corresponds to a wave. The approximate wavelength of the electrons in an electron microscope is about $0.6 \text{ \AA}$ so that one can measure atomic sizes $\approx 1 \text{ \AA}$ using this microscope.

Example 1.3: The moon is at a distance of $3.84 \times 10^8 \text{ m}$ from the Earth. If viewed from two diametrically opposite points on the Earth, the angle subtended at the moon is $1^\circ 54'$. What is the diameter of the Earth?

Solution: Angle subtended

$$\theta = 1^\circ 54' = 114' = 114 \times 2.91 \times 10^{-4} \text{ rad}$$

$$= 3.317 \times 10^{-2} \text{ rad}$$

Diameter of the Earth = $\theta \times$ distance to the moon from the Earth

$$= 3.317 \times 10^{-2} \times 3.84 \times 10^8 \text{ m}$$

$$= 1.274 \times 10^7 \text{ m}$$

1.4 Measurement of Mass:

Since 1889, a kilogram was the mass of a shiny piece of platinum-iridium alloy kept in a special glass case at the International Bureau of weights and measures. This definition of mass has been modified on 20th May 2019, the reason being that the carefully kept platinum-iridium piece is seen to pick up micro particles of dirt and is also affected by the atmosphere causing its mass to change. The new measure of kilogram is defined in terms of magnitude of electric current. We know that electric current can be used to make an electromagnet. An electromagnet attracts magnetic materials and is thus used in research and in industrial applications such as cranes to lift heavy pieces of iron/steel. Thus the kilogram mass can be described in terms of the amount of current which has to be passed through an electromagnet so that it can pull down one side of an extremely sensitive balance to balance the other side which holds one standard kg mass.

While dealing with mass of atoms and molecules, kg is an inconvenient unit. Therefore, their mass is measured in atomic mass unit. It will be easy to compare mass of any atom in terms of mass of some standard atom which has been decided internationally to be C^{12} atom. The $(1/12)^{\text{th}}$ mass of an unexcited atom of C^{12} is called atomic mass unit (amu).

$1 \text{ amu} = 1.6605402 \times 10^{-27} \text{ kg}$ with an uncertainty of 10 in the last two decimal places.

1.5 Measurement of Time:

The SI unit of time is the second (s). For many years, duration of one mean Solar day was considered as reference. A mean Solar day is the average time interval from one noon to the next noon. Average duration of a day is taken as 24 hours. One hour is of 60 minutes

and each minute is of 60 seconds. Thus a mean Solar day = 24 hours = $24 \times 60 \times 60 = 86400$ s. Accordingly a second was defined as $1/86400$ of a mean Solar day.

It was later observed that the length of a Solar day varies gradually due to the gradual slowing down of the Earth's rotation. Hence, to get more standard and nonvarying (constant) unit for measurement of time, a cesium atomic clock is used. It is based on periodic vibrations produced in cesium atom. In cesium atomic clock, a second is taken as the time needed for 9,192,631,770 vibrations of the radiation (wave) emitted during a transition between two hyperfine states of Cs^{133} atom.



Do you know ?

Why is only carbon used and not any other element for defining atomic mass unit? Carbon 12 (C^{12}) is the most abundant isotope of carbon and the most stable one. Around 98% of the available carbon is C^{12} isotope.

Earlier, oxygen and hydrogen were used as the standard atoms. But various isotopes of oxygen and hydrogen are present in higher proportion and therefore it is more accurate to use C^{12} .

1.6 Dimensions and Dimensional Analysis:

As mentioned earlier, a derived physical quantity can be expressed in terms of some combination of seven basic or fundamental

quantities. For convenience, the basic quantities are represented by symbols as 'L' for length, 'M' for mass, 'T' for time, 'K' For temperature, 'I' for current, 'C' for luminous intensity and 'mol' for amount of mass.

The dimensions of a physical quantity are the powers to which the concerned fundamental units must be raised in order to obtain the unit of the given physical quantity.

When we represent any derived quantity with appropriate powers of symbols of the fundamental quantities, then such an expression is called dimensional formula. This dimensional formula is expressed by square bracket and no comma is written in between any of the symbols.

Illustration:

(i) Dimensional formula of velocity

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{Dimensions of velocity} = \frac{[L]}{[T]} = [L^1 M^0 T^{-1}]$$

ii) Dimensional formula of velocity gradient

$$\text{velocity gradient} = \frac{\text{velocity}}{\text{distance}}$$

Dimensions of velocity gradient

$$= \frac{[LT^{-1}]}{[L]} = [L^0 M^0 T^{-1}]$$

iii) Dimensional formula for charge.

$$\text{charge} = \text{current} \times \text{time}$$

$$\text{Dimensions of charge} = [I] [T] = [L^0 M^0 T^1 I^1]$$

Table 1.3: Some Common Physical Quantities their, SI Units and Dimensions

S. No	Physical quantity	Formula	SI unit	Dimensional formula
1	Density	$\rho = M/V$	kilogram per cubic metre (kg/m^3)	$[L^{-3} M^1 T^0]$
2	Acceleration	$a = v/t$	metre per second square (m/s^2)	$[L^1 M^0 T^{-2}]$
3	Momentum	$P = mv$	kilogram metre per second (kg m/s)	$[L^1 M^1 T^{-1}]$
4	Force	$F = ma$	kilogram metre per second square (kg m/s^2) or newton (N)	$[L^1 M^1 T^{-2}]$
5	Impulse	$J = F \cdot t$	newton second (Ns)	$[L^1 M^1 T^{-1}]$
6	Work	$W = F \cdot s$	joule (J)	$[L^2 M^1 T^{-2}]$
7	Kinetic Energy	$KE = 1/2 mv^2$	joule (J)	$[L^2 M^1 T^{-2}]$
8	Pressure	$P = F/A$	kilogram per metre second square (kg/ms^2)	$[L^{-1} M^1 T^{-2}]$

Table 1.3 gives the dimensions of various physical quantities commonly used in mechanics.

1.6.1 Uses of Dimensional Analysis:

- (i) **To check the correctness of physical equations:** In any equation relating different physical quantities, if the dimensions of all the terms on both the sides are the same then that equation is said to be dimensionally correct. This is called the principle of homogeneity of dimensions. Consider the first equation of motion.

$$v = u + at$$

$$\text{Dimension of L.H.S} = [v] = [LT^{-1}]$$

$$[u] = [LT^{-1}]$$

$$[at] = [LT^{-2}] [T] = [LT^{-1}]$$

$$\text{Dimension of R.H.S} = [LT^{-1}] + [LT^{-1}]$$

$$[L.H.S] = [R.H.S]$$

As the dimensions of L.H.S and R.H.S are the same, the given equation is dimensionally correct.

- (ii) **To establish the relationship between related physical quantities:** The period T of oscillation of a simple pendulum depends on length l and acceleration due to gravity g . Let us derive the relation between T, l, g :

$$\text{Suppose } T \propto l^a$$

$$\text{and } T \propto g^b$$

$$\therefore T \propto l^a g^b$$

$$T = k l^a g^b,$$

where k is constant of proportionality and it is a dimensionless quantity and a and b are rational numbers. Equating dimensions on both sides,

$$[L^0 M^0 T^1] = k [L^1]^a [LT^{-2}]^b \\ = k [L^{a+b} T^{-2b}]$$

$$[L^0 T^1] = k [L^{a+b} T^{-2b}]$$

Comparing the dimensions of the corresponding quantities on both the sides we get

$$a + b = 0$$

$$\therefore a = -b$$

and

$$-2b = 1$$

$$\therefore b = -1/2$$

$$\therefore a = -b = -(-1/2)$$

$$\therefore a = 1/2$$

$$\therefore T = k l^{1/2} g^{-1/2}$$

$$\therefore T = k \sqrt{l/g}$$

The value of k is determined experimentally and is found to be 2π

$$\therefore T = 2\pi \sqrt{l/g}$$

- (iii) **To find the conversion factor between the units of the same physical quantity in two different systems of units:** Let us use dimensional analysis to determine the conversion factor between joule (SI unit of work) and erg (CGS unit of work).

$$\text{Let } 1 \text{ J} = x \text{ erg}$$

Dimensional formula for work is $[L^2 M^1 T^{-2}]$
Substituting in the above equation, we can write

$$[L_1^2 M_1^1 T_1^{-2}] = x [L_2^2 M_2^1 T_2^{-2}]$$

$$x = \frac{[L_1^2 M_1^1 T_1^{-2}]}{[L_2^2 M_2^1 T_2^{-2}]}$$

$$\text{or, } x = \left(\frac{L_1}{L_2} \right)^2 \left(\frac{M_1}{M_2} \right)^1 \left(\frac{T_1}{T_2} \right)^{-2}$$

where suffix 1 indicates SI units and 2 indicates CGS units.

In SI units, L, M, T are expressed in m, kg and s and in CGS system L, M, T are represented in cm, g and s respectively.

$$\therefore x = \left(\frac{\text{kg}}{\text{g}} \right)^1 \left(\frac{\text{m}}{\text{cm}} \right)^2 \left(\frac{\text{s}}{\text{s}} \right)^{-2}$$

$$\text{or } x = \left(10^3 \frac{\text{g}}{\text{g}} \right)^1 \left(100 \frac{\text{cm}}{\text{cm}} \right)^2 (1)^{-2}$$

$$\therefore x = (10^3)(10^4) = 10^7$$

$$\therefore 1 \text{ joule} = 10^7 \text{ erg}$$

Example 1.4: A calorie is a unit of heat and it equals 4.2 J, where $1 \text{ J} = \text{kg m}^2 \text{ s}^{-2}$. A distant civilisation employs a system of units in which the units of mass, length and time are $\alpha \text{ kg}, \beta \text{ m}$

and γ s. Also J' is their unit of energy. What will be the magnitude of calorie in their units?

Solution: Let us write the new units as A, B and C for mass, length and time respectively. We are given

$$1 \text{ A} = \alpha \text{ kg}$$

$$1 \text{ B} = \beta \text{ m}$$

$$1 \text{ C} = \gamma \text{ s}$$

$$1 \text{ cal} = 4.2 \text{ J} = 4.2 \text{ kg m}^2 \text{ s}^{-2}$$

$$= 4.2 \left(\frac{\text{A}}{\alpha} \right) \left(\frac{\text{B}}{\beta} \right)^2 \left(\frac{\text{C}}{\gamma} \right)^{-2}$$

$$= \frac{4.2 \gamma^2}{\alpha \beta^2} \text{ AB}^2 \text{ C}^{-2}$$

$$= \frac{4.2 \gamma^2}{\alpha \beta^2} \text{ J'}$$

$$\text{Thus in the new units, } 1 \text{ calorie is } = \frac{4.2 \gamma^2}{\alpha \beta^2} \text{ J'}$$

1.6.2 Limitations of Dimensional Analysis:

- 1) The value of dimensionless constant can be obtained with the help of experiments only.
- 2) Dimensional analysis can not be used to derive relations involving trigonometric, exponential, and logarithmic functions as these quantities are dimensionless.
- 3) This method is not useful if constant of proportionality is not a dimensionless quantity.

Illustration : Gravitational force between two point masses is directly proportional to product of the two masses and inversely proportional to square of the distance between the two

$$\therefore F \propto \frac{m_1 m_2}{r^2}$$

$$\text{Let } F = G \frac{m_1 m_2}{r^2}$$

The constant of proportionality 'G' is NOT dimensionless. Thus earlier method will not work.

- 4) If the correct equation contains some more terms of the same dimension, it is not possible to know about their presence using dimensional equation. For example, with

standard symbols, the equation $S = \frac{1}{2} at^2$ is dimensionally correct. However, the complete equation is $S = ut + \frac{1}{2} at^2$

1.7 Accuracy, Precision and Uncertainty in Measurement:

Physics is a science based on observations and experiments. Observations of various physical quantities are made during an experiment. For example, during the atmospheric study we measure atmospheric pressure, wind velocity, humidity, etc. All the measurements may be accurate, meaning that the measured values are the same as the true values. Accuracy is how close a measurement is to the actual value of that quantity. These measurements may be precise, meaning that multiple measurements give nearly identical values (i.e., reproducible results). In actual measurements, an observation may be both accurate and precise or neither accurate nor precise. The goal of the observer should be to get accurate as well as precise measurements.

Possible uncertainties in an observation may arise due to following reasons:

- 1) Quality of instrument used.
- 2) Skill of the person doing the experiment.
- 3) The method used for measurement.
- 4) External or internal factors affecting the result of the experiment.



Can you tell?

If ten students are asked to measure the length of a piece of cloth up to a mm, using a metre scale, do you think their answers will be identical? Give reasons.

1.8 Errors in Measurements:

Faulty measurements of physical quantity can lead to errors. The errors are broadly divided into the following two categories :

a) Systematic errors : Systematic errors are errors that are not determined by chance but are introduced by an inaccuracy (involving

either the observation or measurement process) inherent to the system. Sources of systematic error may be due to imperfect calibration of the instrument, and sometimes imperfect method of observation.

Each of these errors tends to be in one direction, either positive or negative. The sources of systematic errors are as follows:

- (i) **Instrumental error:** This type of error arises due to defective calibration of an instrument, for example an incorrect zeroing of an instrument will lead to such kind of error ('zero' of a thermometer not graduated at proper place, the pointer of weighing balance in the laboratory already indicating some value instead of showing zero when no load is kept on it, an ammeter showing a current of 0.5 amp even when not connected in circuit, etc).
- (ii) **Error due to imperfection in experimental technique:** This is an error due to defective setting of an instrument. For example the measured volume of a liquid in a graduated tube will be inaccurate if the tube is not held vertical.
- (iii) **Personal error:** Such errors are introduced due to fault of the observer. Bias of the observer, carelessness in taking observations etc. could result in such errors. For example, while measuring the length of an object with a ruler, it is necessary to look at the ruler directly from above. If the observer looks at it from an angle, the measured length will be wrong due to parallax.

Systematic errors can be minimized by using correct instrument, following proper experimental procedure and removing personal error.

b) Random errors: These are the errors which are introduced even after following all the procedures to minimize systematic errors. These type of errors may be positive or negative. These errors can not be eliminated completely but we can minimize them by repeated observations and then taking their mean (average). Random errors occur due to variation in conditions in

which experiment is performed. For example, the temperature may change during the course of an experiment, pressure of any gas used in the experiment may change, or the voltage of the power supply may change randomly, etc.

1.8.1 Estimation of error:

Suppose the readings recorded repeatedly for a physical quantity during a measurement are

$$a_1, a_2, a_3, \dots, a_n.$$

Arithmetic mean a_{mean} is given by

$$a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

$$a_{\text{mean}} = \frac{1}{n} \sum_{i=1}^n a_i \quad \text{--- (1.3)}$$

This is the most probable value of the quantity. The magnitude of the difference between mean value and each individual value is called **absolute error** in the observation.

Thus for ' a_1 ', the absolute error Δa_1 is given by

$$\Delta a_1 = |a_{\text{mean}} - a_1|,$$

for a_2 ,

$$\Delta a_2 = |a_{\text{mean}} - a_2|$$

and so for a_n it will be

$$\Delta a_n = |a_{\text{mean}} - a_n|$$

The arithmetic mean of all the absolute errors is called **mean absolute error** in the measurement of the physical quantity.

$$\Delta a_{\text{mean}} = \frac{\Delta a_1 + \Delta a_2 + \dots + \Delta a_n}{n}$$

$$= \frac{1}{n} \sum_{i=1}^n \Delta a_i \quad \text{--- (1.4)}$$

The measured value of the physical quantity a can then be represented by

$a = a_{\text{mean}} \pm \Delta a_{\text{mean}}$ which tells us that the actual value of ' a ' could be between $a_{\text{mean}} - \Delta a_{\text{mean}}$ and $a_{\text{mean}} + \Delta a_{\text{mean}}$. The ratio of mean absolute error to its arithmetic mean value is called **relative error**.

$$\text{Relative error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \quad \text{--- (1.5)}$$

When relative error is represented as percentage it is called **percentage error**.

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100 \quad \text{--- (1.6)}$$



Activity :

Perform an experiment using a Vernier callipers of least count 0.01cm to measure the external diameter of a hollow cylinder. Take 3 readings at different position on the cylinder and find (i) the mean diameter (ii) the absolute mean error and (iii) the percentage error in the measurement of diameter.

Example 1.5: The radius of a sphere measured repeatedly yields values 5.63m, 5.54 m, 5.44 m, 5.40 m and 5.35 m. Determine the most probable value of radius and the mean absolute, relative and percentage errors.

Solution: Most probable value of radius is its arithmetic mean

$$= \frac{5.63 + 5.54 + 5.44 + 5.40 + 5.35}{5} \text{ m}$$

$$= 5.472 \text{ m}$$

Mean absolute error

$$= \frac{1}{5} \left\{ \begin{array}{l} |5.63 - 5.472| + |5.54 - 5.472| \\ + |5.44 - 5.472| + |5.40 - 5.472| \\ + |5.35 - 5.472| \end{array} \right\} \text{ m}$$

$$= \frac{0.452}{5} = 0.0904 \text{ m}$$

$$\text{Relative error} = \frac{0.0904}{5.472} = 0.017$$

$$\% \text{ error} = 1.7\%$$

1.8.2 Combination of errors:

When we do an experiment and measure various physical quantities associated with the experiment, we must know how the errors from individual measurement combine to give errors in the final result. For example, in the measurement of the resistance of a conductor using Ohms law, there will be an error in the measurement of potential difference and that of current. It is important to study how these errors combine to give the error in the measurement of

resistance.

a) Errors in sum and in difference:

Suppose two physical quantities A and B have measured values $A \pm \Delta A$ and $B \pm \Delta B$, respectively, where ΔA and ΔB are their mean absolute errors. We wish to find the absolute error ΔZ in their sum.

$$Z = A + B$$

$$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$

$$= (A + B) \pm \Delta A \pm \Delta B$$

$$\pm \Delta Z = \pm \Delta A \pm \Delta B,$$

For difference, i.e., if $Z = A - B$,

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$

$$= (A - B) \pm \Delta A \mp \Delta B$$

$$\pm \Delta Z = \pm \Delta A \pm \Delta B,$$

There are four possible values for ΔZ , namely $(+ \Delta A - \Delta B)$, $(+\Delta A + \Delta B)$, $(-\Delta A - \Delta B)$, $(-\Delta A + \Delta B)$. Hence maximum value of absolute error is $\Delta Z = \Delta A + \Delta B$ in both the cases.

When two quantities are added or subtracted, the maximum absolute error in the final result is the *sum* of the absolute errors in the individual quantities.

b) Errors in product and in division:

Suppose $Z = AB$ and measured values of A and B are $(A \pm \Delta A)$ and $(B \pm \Delta B)$ Then

$$Z \pm \Delta Z = (A \pm \Delta A) (B \pm \Delta B)$$

$$= AB \pm A\Delta B \pm B\Delta A \pm \Delta A\Delta B$$

Dividing L.H.S by Z and R.H.S. by AB we get

$$\left(1 \pm \frac{\Delta Z}{Z} \right) = \left(1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \left(\frac{\Delta A}{A} \right) \left(\frac{\Delta B}{B} \right) \right)$$

Since $\Delta A/A$ and $\Delta B/B$ are very small we shall neglect their product. Hence maximum *relative* error in Z is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B} \quad \text{--- (1.7)}$$

This formula also applies to the division of two quantities.

Thus, when two quantities are multiplied or divided, the maximum relative error in the result is the sum of *relative* errors in each quantity.

c) Errors due to the power (index) of measured quantity:

Suppose

$$Z = A^3 = A.A.A$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta A}{A} + \frac{\Delta A}{A}$$

from the multiplication rule above.

Hence the maximum relative error in $Z = A^3$ is three times the relative error in A . So if $Z = A^n$

$$\frac{\Delta Z}{Z} = n \frac{\Delta A}{A} \quad \text{--- (1.8)}$$

In general, if $Z = \frac{A^p B^q}{C^r}$

$$\frac{\Delta Z}{Z} = p \frac{\Delta A}{A} + q \frac{\Delta B}{B} + r \frac{\Delta C}{C} \quad \text{--- (1.9)}$$

The quantity in the formula which has large power is responsible for maximum error.

Example 1.6: In an experiment to determine the volume of an object, mass and density are recorded as $m = (5 \pm 0.15)$ kg and $\rho = (5 \pm 0.2)$ kg m⁻³ respectively. Calculate percentage error in the measurement of volume.

Solution : We know,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\therefore \text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{M}{\rho}$$

Maximum percentage error in volume

$$\begin{aligned} &= \left(\frac{\Delta m}{m} + \frac{\Delta \rho}{\rho} \right) \times 100 \\ &= \left(\frac{0.15}{5} + \frac{0.2}{5} \right) \times 100 \\ &= (0.03 + 0.04) \times 100 \\ &= (0.07) \times 100 = 7\% \end{aligned}$$

Example 1.7: The acceleration due to gravity is determined by using a simple pendulum of length $l = (100 \pm 0.1)$ cm. If its time period is $T = (2 \pm 0.01)$ s, find the maximum percentage error in the measurement of g .

Solution: The time period of oscillation of a simple pendulum is given by

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Squaring both sides

$$T^2 = 4\pi^2 l / g$$

$$\therefore g = 4\pi^2 \frac{l}{T^2}$$

$$\text{Now } \Delta l = 0.1, l = 100 \text{ cm}, \Delta T = 0.01 \text{ s}, T = 2 \text{ s}$$

$$\begin{aligned} \text{Maximum Percentage error} &= \frac{\Delta g \times 100}{g} \\ &= \left(\frac{\Delta l}{l} + \frac{2\Delta T}{T} \right) \times 100 \\ &= \left(\frac{0.1}{100} + \frac{2 \times 0.01}{2} \right) \times 100 \\ &= (0.001 + 0.01) \times 100 = 1.1 \end{aligned}$$

Maximum percentage error in measurement of g is 1.1%

1.9 Significant Figures:

In the previous sections, we have studied various types of errors, their origins and the ways to minimize them. Our accuracy is limited to the least count of the instrument used during the measurement. Least count is the smallest measurement that can be made using the given instrument. For example with the usual metre scale, one can measure 0.1 cm as the least value. Hence its least count is 0.1 cm.

Suppose we measure the length of a metal rod using a metre scale of least count 0.1 cm. The measurement is done three times and the readings are 15.4, 15.4, and 15.5 cm. The most probable length which is the arithmetic mean as per our earlier discussion is 15.43. Out of this we are certain about the digits 1 and 5 but are not certain about the last 2 digits because of the least count limitation.

The number of digits in a measurement about which we are certain, plus one additional digit, the first one about which we are not certain is known as significant figures or significant digits.

Thus in above example, we have 3 significant digits 1, 5 and 4.

The larger the number of significant figures obtained in a measurement, the greater is the accuracy of the measurement. If one uses the instrument of smaller least count, the number of significant digits increases.

Rules for determining significant figures

- 1) All the nonzero digits are significant, for example if the volume of an object is 178.43 cm^3 , there are five significant digits which are 1, 7, 8, 4 and 3.
- 2) All the zeros between two nonzero digits are significant, eg., $m = 165.02 \text{ g}$ has 5 significant digits.
- 3) If the number is less than 1, the zero/zeros on the right of the decimal point and to the left of the first nonzero digit are not significant e.g. in 0.001405, the underlined zeros are not significant. Thus the above number has four significant digits.
- 4) The zeros on the right hand side of the last nonzero number are significant (but for this, the number must be written with a decimal point), e.g. 1.500 or 0.01500 have both 4 significant figures each.

On the contrary, if a measurement yields length L given as

$L = 125 \text{ m} = 12500 \text{ cm} = 125000 \text{ mm}$, it has only three significant digits.

To avoid the ambiguities in determining the number of significant figures, it is necessary to report every measurement in scientific notation (i.e., in powers of 10) i.e., by using the concept of order of magnitude.

The magnitude of any physical quantity can be expressed as $A \times 10^n$ where 'A' is a number such that $0.5 \leq A < 5$ and 'n' is an integer called the **order of magnitude**.

(i) radius of Earth = 6400 km
 $= 0.64 \times 10^7 \text{ m}$

The order of magnitude is 7 and the number of significant figures are 2.

(ii) Magnitude of the charge on electron e
 $= 1.6 \times 10^{-19} \text{ C}$

Here the order of magnitude is -19 and the number of significant digits are 2.



Internet my friend

1. videlectures.net/mit801f99_lewin lec01/
2. hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

Definitions of SI Units

Till May 20, 2019 the *kilogram* did not have a definition; it was mass of the prototype cylinder kept under controlled conditions of temperature and pressure at the SI museum at Paris. A rigorous and meticulous experimentation has shown that the mass of the *standard* prototype for the *kilogram* has changed in the course of time. This shows the acute necessity for standardisation of units. The new definitions aim to improve the SI without changing the size of any units, thus ensuring continuity with existing measurements. In November 2018, the 26th General Conference on Weights and Measures (CGPM) unanimously approved these changes, which the International Committee for Weights and Measures (CIPM) had proposed earlier that year. These definitions came in force from 20 May 2019.

- (i) As per new SI units, each of the seven fundamental units (metre, kilogram, etc.) uses **one** of the following 7 constants which are proposed to be having **exact values** as given below:

The Planck constant,
 $h = 6.62607015 \times 10^{-34} \text{ joule-second}$
(J s **or** $\text{kg m}^2 \text{s}^{-1}$).

The elementary charge,
 $e = 1.602176634 \times 10^{-19} \text{ coulomb}$ (**C or A s**).

The Boltzmann constant,
 $k = 1.380649 \times 10^{-23} \text{ joule per kelvin}$
(J K^{-1} **or** $\text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$).

The Avogadro constant (number),
 $N_A = 6.02214076 \times 10^{23} \text{ reciprocal mole}$
(mol^{-1}).

The speed of light in vacuum,
 $c = 299792458 \text{ metre per second}$ (m s^{-1}).

The ground state hyperfine structure transition frequency of Caesium-133 atom,

$$\Delta\nu_{\text{Cs}} = 9192631770 \text{ hertz (Hz or s}^{-1}\text{)}.$$

The luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12} \text{ Hz}$, $K_{\text{cd}} = 683 \text{ lumen per watt}$ ($\text{lm} \cdot \text{W}^{-1}$) = $683 \text{ cd sr s}^3 \text{ kg}^{-1} \text{ m}^{-2}$, where sr is steradian; the SI unit of solid angle.

- (ii) Definitions of the units *second* and *mole* are based only upon their respective constants, for example (a) the *second* uses ground state hyperfine structure transition frequency of Caesium-133 atom to be exactly 9192631770 hertz. Thus, the *second* is defined as 9192631770 periods of that transition, (b) the *mole* uses Avogadro's number to be $N_A = 6.02214076 \times 10^{23}$. Thus, one *mole* is that amount of substance which contains exactly $6.02214076 \times 10^{23}$ molecules.
- (iii) Definitions of all the other fundamental units use one constant each and at least one other fundamental unit, for example, the *metre* makes use of speed of light in vacuum as a constant and *second* as fundamental unit. Thus, *metre* is defined as the distance traveled by light in vacuum in exactly $1/299792458$ *second*.
- (iv) The figures show the dependency of various units upon their respective constants and other units (wherever

used). The arrows arriving at that unit refer to the constant and the fundamental unit (or units, wherever used) for defining that unit. The arrows going away from a unit indicate other units which use this unit for their definition.

For example, as described above, in Fig (a), i) the arrows directed to *metre* are from *second* and *c*. The arrows going away from the *metre* indicate that the *metre* is used in defining the *kilogram* the *candela* and the *kelvin*, (ii) the newly defined unit *kilogram* uses Planck constant, the *metre* and the *second*, while the *kilogram* itself is used in defining the *kelvin* and the *candela*. This definition relates the *kilogram* to the equivalent mass of the energy of a photon given its frequency, via the Planck constant.

Fig (a) refers to new definitions while the Fig (b) refers to the corresponding definitions before 20 May 2019. Interested students may compare them to know which definitions are modified and how.

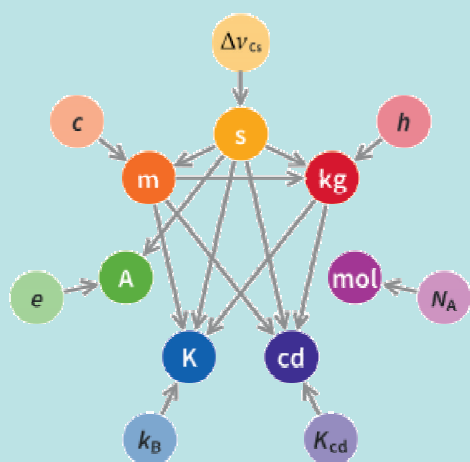


Fig (a) New SI

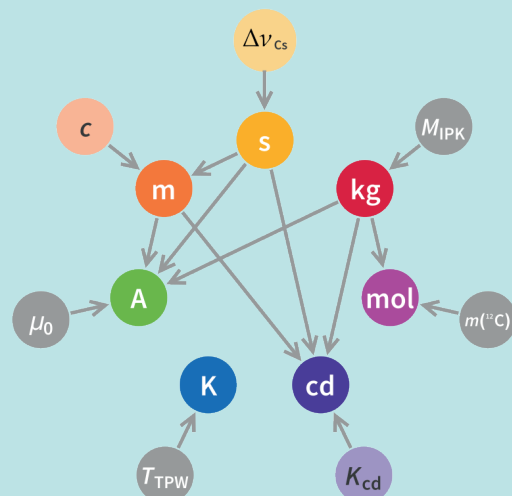


Fig (b) Old SI



Exercises

1. Choose the correct option.

- $[L^1M^1T^{-2}]$ is the dimensional formula for
(A) Velocity (B) Acceleration
(C) Force (D) Work
- The error in the measurement of the sides of a rectangle is 1%. The error in the measurement of its area is
(A) 1% (B) $(\frac{1}{2})\%$
(C) 2% (D) None of the above.
- Light year is a unit of
(A) Time (B) Mass
(C) Distance (D) Luminosity
- Dimensions of kinetic energy are the same as that of
(A) Force (B) Acceleration
(C) Work (D) Pressure
- Which of the following is not a fundamental unit?
(A) cm (B) kg
(C) centigrade (D) volt

2. Answer the following questions.

- Star A is farther than star B. Which star will have a large parallax angle?
- What are the dimensions of the quantity $l\sqrt{l/g}$, l being the length and g the acceleration due to gravity?
- Define absolute error, mean absolute error, relative error and percentage error.
- Describe what is meant by significant figures and order of magnitude.
- If the measured values of two quantities are $A \pm \Delta A$ and $B \pm \Delta B$, ΔA and ΔB being the mean absolute errors. What is the maximum possible error in $A \pm B$? Show that if $Z = \frac{A}{B}$
$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$
- Derive the formula for kinetic energy of a particle having mass m and velocity v using dimensional analysis

3. Solve numerical examples.

- The masses of two bodies are measured to be 15.7 ± 0.2 kg and 27.3 ± 0.3 kg. What is the total mass of the two and the error in it?
[Ans : 43 kg, ± 0.5 kg]
- The distance travelled by an object in time (100 ± 1) s is (5.2 ± 0.1) m. What is the speed and its relative error?
[Ans : 0.052 m s^{-1} , $\pm 0.0292 \text{ m s}^{-1}$]
- An electron with charge e enters a uniform magnetic field $\vec{B}$ with a velocity $\vec{v}$. The velocity is perpendicular to the magnetic field. The force on the charge e is given by $|\vec{F}| = Bev$. Obtain the dimensions of $\vec{B}$.
[Ans: $[L^0M^1T^{-2}I^{-1}]$]
- A large ball 2 m in radius is made up of a rope of square cross section with edge length 4 mm. Neglecting the air gaps in the ball, what is the total length of the rope to the nearest order of magnitude?
[Ans : $\approx 10^6 \text{ m} = 10^3 \text{ km}$]
- Nuclear radius R has a dependence on the mass number (A) as $R = 1.3 \times 10^{-16} A^{1/3} \text{ m}$. For a nucleus of mass number $A=125$, obtain the order of magnitude of R expressed in metre.
[Ans : -15]
- In a workshop a worker measures the length of a steel plate with a Vernier callipers having a least count 0.01 cm. Four such measurements of the length yielded the following values: 3.11 cm, 3.13 cm, 3.14 cm, 3.14 cm. Find the mean length, the mean absolute error and the percentage error in the measured value of the length.
[Ans: 3.13 cm, 0.01 cm, 0.32%]

- vii) Find the percentage error in kinetic energy of a body having mass 60.0 ± 0.3 g moving with a velocity 25.0 ± 0.1 cm/s.
[Ans: 1.3%]
- viii) In Ohm's experiments, the values of the unknown resistances were found to be 6.12Ω , 6.09Ω , 6.22Ω , 6.15Ω . Calculate the mean absolute error, relative error and percentage error in these measurements.
[Ans: 0.04Ω , 0.0065Ω , 0.65%]
- ix) An object is falling freely under the gravitational force. Its velocity after travelling a distance h is v . If v depends upon gravitational acceleration g and distance, prove with dimensional analysis that $v = k\sqrt{gh}$ where k is a constant.
- x) $v = at + \frac{b}{t+c} + v_0$ is a dimensionally valid equation. Obtain the dimensional formula for a , b and c where v is velocity, t is time and v_0 is initial velocity.
[Ans: a - $[L^1 M^0 T^{-2}]$, b - $[L^1 M^0 T^0]$, c - $[L^0 M^0 T^1]$]
- xi) The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.
[Ans: 4.255 m^2 , 8.552 m^3]
- xii) If the length of a cylinder is $l = (4.00 \pm 0.001)$ cm, radius $r = (0.0250 \pm 0.001)$ cm and mass $m = (6.25 \pm 0.01)$ gm. Calculate the percentage error in the determination of density.
[Ans: 8.185%]
- xiii) When the planet Jupiter is at a distance of 824.7 million kilometer from the Earth, its angular diameter is measured to be $35.72''$ of arc. Calculate the diameter of the Jupiter.
[Ans: $1.428 \times 10^5 \text{ km}$]
- xiv) If the formula for a physical quantity is $X = \frac{a^4 b^3}{c^{1/3} d^{1/2}}$ and if the percentage error in the measurements of a , b , c and d are 2%, 3%, 3% and 4% respectively. Calculate percentage error in X .
[Ans: 20%]
- xv) Write down the number of significant figures in the following: 0.003 m^2 , $0.1250 \text{ gm cm}^{-2}$, $6.4 \times 10^6 \text{ m}$, $1.6 \times 10^{-19} \text{ C}$, $9.1 \times 10^{-31} \text{ kg}$.
[Ans: 1, 4, 2, 2, 2]
- xvi) The diameter of a sphere is 2.14 cm. Calculate the volume of the sphere to the correct number of significant figures.
[Ans: 5.13 cm^3]



Can you recall?

1. What is the difference between a scalar and a vector?
2. Which of the following are scalars or vectors?
 - (i) displacement (ii) distance travelled (iii) velocity
 - (iv) speed (v) force (vi) work done (vii) energy

2.1 Introduction:

You will need certain mathematical tools to understand the topics covered in this book. Vector analysis and elementary calculus are two among these. You will learn calculus in details, in mathematics, in the XIIth standard. In this Chapter, you are going to learn about vector analysis and will have a preliminary introduction to calculus which should be sufficient for you to understand the physics that you will learn in this book.

2.2 Vector Analysis:

In the previous Chapter, you have studied different aspects of physical quantities like their division into fundamental and derived quantities and their units and dimensions. You also need to understand that all physical quantities may not be fully described by their magnitudes and units alone. For example if you are given the time for which a man has walked with a certain speed, you can find the distance travelled by the man, but you cannot find out where exactly the man has reached unless you know the direction in which the man has walked.

Therefore, you can say that some physical quantities, which are called scalars, can be described with magnitude alone, whereas some other physical quantities, which are called vectors, need to be described with magnitude as well as direction. In the above example the distance travelled by the man is a scalar quantity while the final position of the man relative to his initial position, i.e., his displacement can be described by magnitude and direction and is a vector quantity. In this Chapter you will study different aspects of scalar and vector quantities.

2.2.1 Scalars:

Physical quantities which can be completely

described by their magnitude are called scalars, i.e. they are specified by a number and a unit. For example when we say that a given metal rod has a length 2 m, it indicates that the rod is two times longer than a certain standard unit *metre*. Thus the number 2 is the magnitude and metre is the unit; together they give us a complete idea about the length of the rod. Thus length is a scalar quantity. Similarly mass, time, temperature, density, etc., are examples of scalars. Scalars can be added or subtracted by rules of simple algebra.

2.2.2 Vectors:

Physical quantities which need magnitude as well as direction for their complete description are called vectors. Examples of vectors are displacement, velocity, force etc.

A vector can be represented by a directed line segment or by an arrow. The length of the line segment drawn to scale gives the magnitude of the vector, e.g., displacement of a body from P to Q can be represented as $P \longrightarrow Q$, where the starting point P is called the tail and the end point Q (arrow head) is called the head of the vector. Symbolically we write it as $\overrightarrow{PQ}$. Symbolically vectors are also represented by a single capital letter with an arrow above it, e.g., $\vec{X}$, $\vec{A}$, etc. Magnitude of a vector $\vec{X}$ is written as $|\vec{X}|$.

Let us see a few examples of different types of vectors.

(a) Zero vector (Null vector): A vector having zero magnitude with a particular direction (arbitrary) is called zero vector. Symbolically it is represented as $\vec{0}$.

(1) Velocity vector of a stationary particle is a zero vector.

(2) The acceleration vector of an object

moving with uniform velocity is a zero vector.

- (b) **Resultant vector:** The resultant of two or more vectors is that single vector, which produces the same effect, as produced by all the vectors together.
- (c) **Negative vector (opposite vector):** A negative vector of a given vector is a vector of the same magnitude but opposite in direction to that of the given vector.

In Fig. 2.1, $\vec{B}$ is a negative vector to $\vec{A}$.

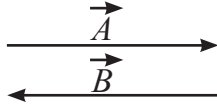


Fig. 2.1: Negative vector.

- (d) **Equal vector:** Two vectors A and B representing same physical quantity are said to be equal if and only if they have the same magnitude and direction. Two equal vectors are shown in Fig. 2.2.

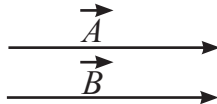


Fig. 2.2: Equal vectors.

- (e) **Position vector:** A vector which gives the position of a particle at a point with respect to the origin of a chosen co-ordinate system is called the position vector of the particle.

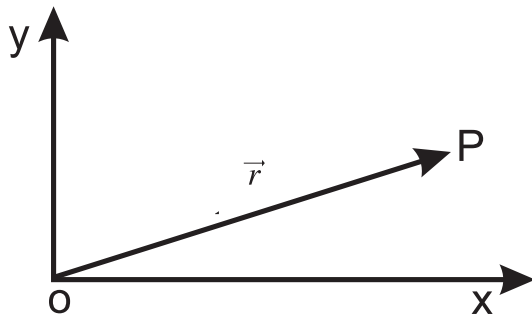


Fig 2.3: Position vector.

In Fig 2.3, $\vec{r} = \vec{OP}$ is the position vector of the particle present at P.

- (f) **Unit vector:** A vector having unit magnitude in a given direction is called a unit vector in that direction. If $\vec{M}$ is a non-zero vector i.e. its magnitude $M = |\vec{M}|$ is not zero, the unit vector along

$\vec{M}$ is written as $\hat{u}_M$ and is given by

$$\vec{M} = \hat{u}_M M \quad \text{--- (2.1)}$$

$$\text{or, } \hat{u}_M = \frac{\vec{M}}{M} \quad \text{--- (2.2)}$$

Hence $\hat{u}_M$ has magnitude unity and has the same direction as that of $\vec{M}$. We use $\hat{i}$, $\hat{j}$, and $\hat{k}$, respectively, as unit vectors along the x, y and z directions of a rectangular (three dimensional) coordinate system.

$$\begin{aligned} \hat{u}_x = \hat{i}, \hat{u}_y = \hat{j} \text{ and } \hat{u}_z = \hat{k} \\ \therefore \hat{i} = \frac{\vec{x}}{x}, \hat{j} = \frac{\vec{y}}{y} \text{ and } \hat{k} = \frac{\vec{z}}{z} \end{aligned} \quad \text{--- (2.3)}$$

Here $\vec{x}$, $\vec{y}$ and $\vec{z}$ are vectors along x, y and z axes, respectively.

2.3 Vector Operations:

2.3.1 Multiplication of a Vector by a Scalar:

Multiplying a vector $\vec{P}$ by a scalar quantity, say s , yields another vector. Let us write

$$\vec{Q} = s\vec{P} \quad \text{--- (2.4)}$$

$\vec{Q}$ will be a vector whose direction is the same as that of $\vec{P}$ and magnitude is s times the magnitude of $\vec{P}$.

2.3.2 Addition and Subtraction of Vectors:

The addition or subtraction of two or more vectors of the same type, i.e., describing the same physical quantity, gives rise to a single vector, such that the effect of this single vector is the same as the net effect of the vectors which have been added or subtracted.

It is important to understand that only vectors of the same type (describing same physical quantity) can be added or subtracted e.g. force $\vec{F}_1$ and force $\vec{F}_2$ can be added to give the resultant force $\vec{F} = \vec{F}_1 + \vec{F}_2$. But a force vector can not be added to a velocity vector.

It is easy to find addition of vectors $\vec{AB}$ and $\vec{BC}$ having the same or opposite direction but different magnitudes. If individual vectors are parallel (i.e., in the same direction), the magnitude of their resultant is the addition of individual magnitudes, i.e., $|\vec{AC}| = |\vec{AB}| + |\vec{BC}|$

and direction of the resultant is the same as that of the individual vectors as shown in Fig 2.4 (a). However, if the individual vectors are anti-parallel (i.e., in the opposite direction), the magnitude of their resultant is the difference of the individual magnitudes, and the direction is that of the larger vector i.e., $|\vec{AC}| = ||\vec{AB}| - |\vec{BC}||$ as shown in Fig. 2.4 (b).

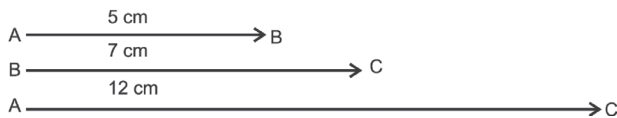


Fig. 2.4 (a): Resultant of parallel displacements.



Fig 2.4 (b): Resultant of anti-parallel forces.

2.3.3 Triangle Law for Vector Addition:

When vectors of a given type do not act in the same or opposite directions, the resultant can be determined by using the triangle law of vector addition which is stated as follows:

If two vectors describing the same physical quantity are represented in magnitude and direction by the two sides of a triangle taken in order, then their resultant is represented in magnitude and direction by the third side of the triangle drawn in the opposite sense (from the starting point of first vector to the end point of the second vector).

Let $\vec{A}$ and $\vec{B}$ be two vectors in the plane of paper as shown in Fig. 2.5 (a). The sum of these two vectors can be obtained by using the triangle law described above as shown in Fig. 2.5 (b). The resultant vector is indicated by $\vec{C}$.

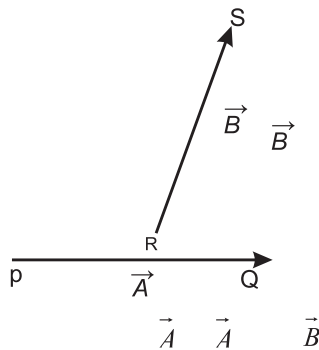


Fig. 2.5 (a): Two vectors $\vec{A}$ and $\vec{B}$ in a plane,

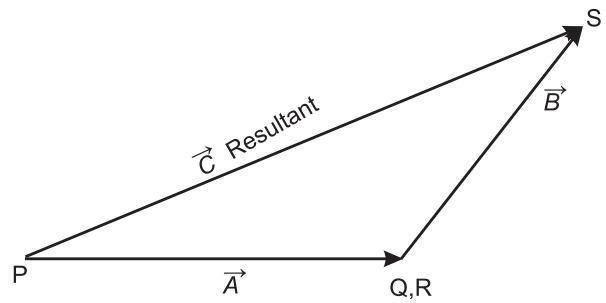


Fig. 2.5 (b): Resultant vector $\vec{C} = \vec{A} + \vec{B}$.

We can use the triangle law for showing that

(a) Vector addition is commutative.

For any two vectors $\vec{P}$ and $\vec{Q}$,

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P} \quad \text{--- (2.5)}$$

Figure 2.6 (a) shows addition of the two vector $\vec{P}$ and $\vec{Q}$ in two different ways. Triangle OAB shows $\vec{P} + \vec{Q} = \vec{R} = \vec{OB}$, while triangle OCB shows $\vec{Q} + \vec{P} = \vec{R} = \vec{OB}$.

$$\therefore \vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

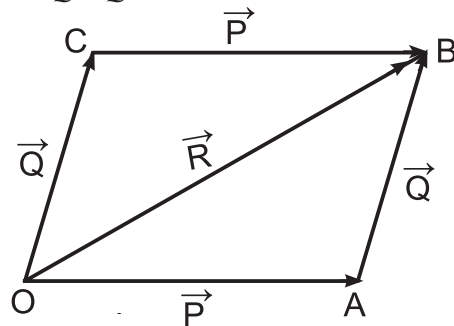


Fig. 2.6 (a): Commutative law.

(b) Vector addition is associative

If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are three vectors then

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

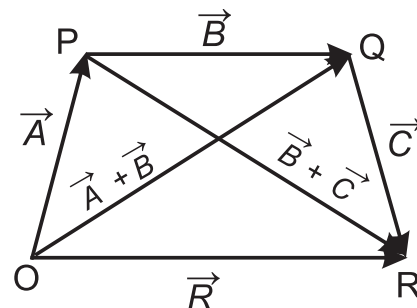


Fig. 2.6 (b): Associative law.

Figure 2.6 (b) shows addition of 3 vectors

$\vec{A}$, $\vec{B}$ and $\vec{C}$ in two different ways to give resultant $\vec{R}$.

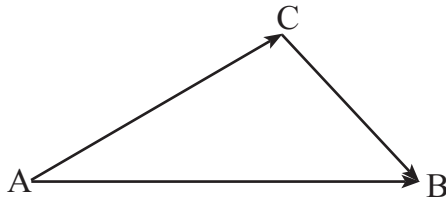
$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} \quad \text{--- from triangle OQR}$$

$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) \quad \text{--- from triangle OPR}$$

$$\text{i.e., } (\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}) \quad \text{--- (2.6)}$$

Thus the Associative law is proved.

Example 2.1: Express vector $\vec{AC}$ in terms of vectors $\vec{AB}$ and $\vec{CB}$ shown in the following figure.



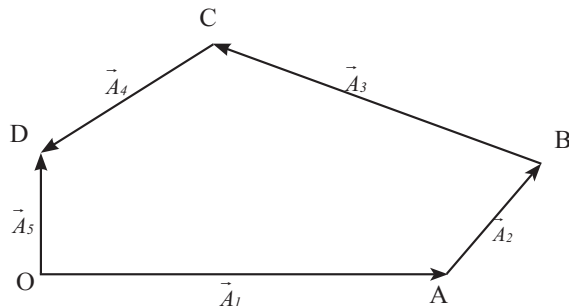
Solution: Using the triangle law of addition of vectors we can write

$$\vec{AC} + \vec{CB} = \vec{AB}$$

$$\therefore \vec{AC} = \vec{AB} - \vec{CB}$$

Example 2.2: From the following figure, determine the resultant of four forces

$$\vec{A}_1, \vec{A}_2, \vec{A}_3 \text{ and } \vec{A}_4$$



Solution: Join $\vec{OB}$ to complete Δ OAB as shown in (a)

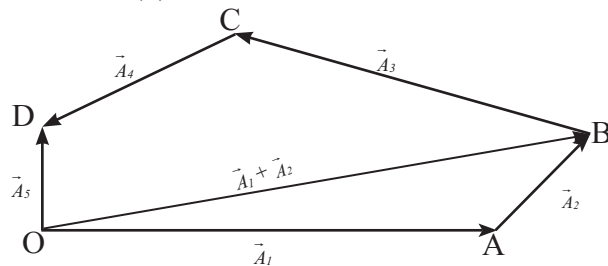


Fig. (a)

$$\text{Now, } \vec{OB} = \vec{OA} + \vec{AB} = \vec{A}_1 + \vec{A}_2$$

Join $\vec{OC}$ to complete triangle OBC as shown in (b).

$$\text{Now, } \vec{OC} = \vec{OB} + \vec{BC} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3$$

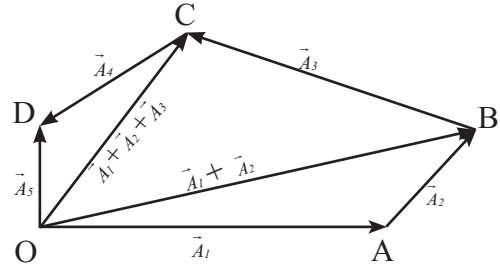


Fig. (b)

From triangle OCD,

$$\vec{OD} = \vec{A}_5 = \vec{OC} + \vec{CD} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \vec{A}_4$$

Thus $\vec{OD}$ is the resultant of the four vectors, $\vec{A}_1, \vec{A}_2, \vec{A}_3$ and $\vec{A}_4$, represented by

$$\vec{OA}, \vec{AB}, \vec{BC} \text{ and } \vec{CD}, \text{ respectively.}$$

2.3.4 Law of parallelogram of vectors:

Another geometrical method of adding two vectors is called parallelogram law of vector addition which is stated as follows:

If two vectors of the same type, originating from the same point (tails at the same point) are represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant vector is given in magnitude and direction by the diagonal of the parallelogram starting from the same point as shown in Fig. 2.7.

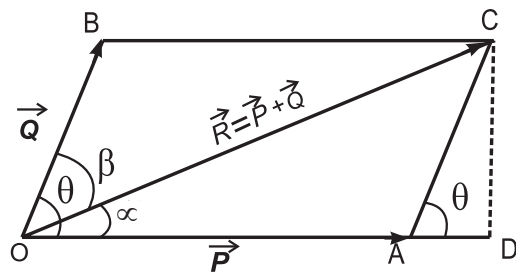


Fig 2.7: Parallelogram law of vector addition.

In Fig. 2.7, vector $\vec{OA} = \vec{P}$ and vector $\vec{OB} = \vec{Q}$, represent two vectors originating from point O, inclined to each other at an angle θ . If we complete the parallelogram, then according to this law, the diagonal $\vec{OC} = \vec{R}$ represents the resultant vector.

To find the magnitude of $\vec{R}$, drop a

perpendicular from C to reach OA (extended) at D. In right angled triangle ODC, by application by Pythagoras theorem,

$$\begin{aligned} OC^2 &= OD^2 + DC^2 \\ &= (OA + AD)^2 + DC^2 \\ OC^2 &= OA^2 + 2OA \cdot AD + AD^2 + DC^2 \end{aligned}$$

In the right angled triangle ADC, by application of Pythagoras theorem

$$\begin{aligned} AD^2 + DC^2 &= AC^2 \\ \therefore OC^2 &= OA^2 + 2OA \cdot AD + AC^2 \quad \text{--- (2.7)} \end{aligned}$$

Also,

$$\vec{OA} = \vec{P}, \vec{AC} = \vec{OB} = \vec{Q} \text{ and } \vec{OC} = \vec{R}$$

In ΔADC , $\cos \theta = AD/AC$

$$\therefore AD = AC \cos \theta = Q \cos \theta$$

Substituting in Eq. (2.7)

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta.$$

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \quad \text{--- (2.8)}$$

Equation (2.8) gives us the magnitude of resultant vector $\vec{R}$.

To find the direction of the resultant vector $\vec{R}$, we will have to find the angle (α) made by $\vec{R}$ with $\vec{P}$.

$$\begin{aligned} \text{In } \Delta ODC, \tan \alpha &= \frac{DC}{OD} \\ &= \frac{DC}{OA + AD} \quad \text{--- (2.9)} \end{aligned}$$

$$\text{From the figure, } \sin \theta = \frac{DC}{AC}$$

$$\therefore DC = AC \sin \theta = Q \sin \theta$$

Also,

$$AD = AC \cos \theta = Q \cos \theta$$

$$\text{and } OA = \vec{P},$$

Substituting in Eq. (2.9), we get

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\therefore \alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right) \quad \text{--- (2.10)}$$

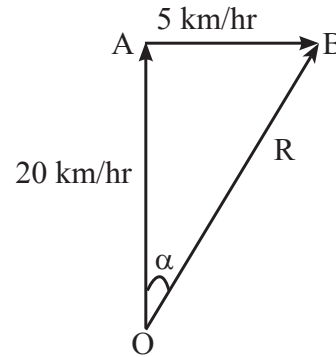
Equation (2.10) gives us the direction of resultant vector $\vec{R}$.

If β is the angle between $\vec{R}$ and $\vec{Q}$, it can be

$$\text{similarly derived that } \beta = \tan^{-1} \left(\frac{P \sin \theta}{Q + P \cos \theta} \right)$$

Example 2.3: Water is flowing in a stream with velocity 5 km/hr in an easterly direction relative to the shore. Speed of a boat is relative to still water is 20 km/hr. If the boat enters the stream heading North, with what velocity will the boat actually travel?

Solution: The resultant velocity $\vec{R}$ of the boat can be obtained by adding the two velocities using ΔOAB shown in the figure. Magnitude of the resultant velocity is calculated as follows:



$$\begin{aligned} R &= \sqrt{20^2 + 5^2} \\ &= \sqrt{425} = 20.61 \text{ km/hr} \end{aligned}$$

The direction of the resultant velocity is

$$= \tan^{-1} \left(\frac{5}{20} \right) = \tan^{-1}(0.25)$$

$$= 14^\circ 04'$$

$\therefore$

The velocity of the boat is 20.61 km/hr in a direction $14^\circ 04'$ east of north.

2.4 Resolution of vectors:

A vector can be written as a sum of two or more vectors along certain fixed directions.

Thus a vector $\vec{V}$ can be written as

$$\vec{V} = V_1 \hat{\alpha} + V_2 \hat{\beta} + V_3 \hat{\gamma} \quad \text{--- (2.11)}$$

where $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ are unit vectors along chosen directions. V_1, V_2 and V_3 are known as components of $\vec{V}$ along the three directions $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$.

The process of splitting a given vector into its components is called resolution of the vector. The components can be found along

directions at any required angles, but if these components are found along the directions which are mutually perpendicular, they are called rectangular components.

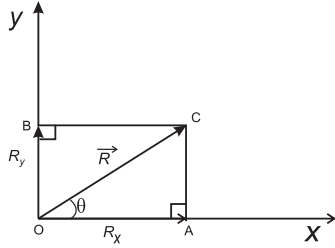


Fig. 2.8 : Resolution of a vector.

Let us see how to find rectangular components in two dimensions.

Consider a vector $\vec{R} = \vec{OC}$, originating from the origin of a rectangular co-ordinate system as shown in Fig. 2.8.

Drop perpendiculars from C that meet the x-axis at A and y-axis at B.

$\vec{OA} = \vec{R}_x$ and $\vec{OB} = \vec{R}_y$; $\vec{R}_x$ and $\vec{R}_y$ being the components of $\vec{OC}$ along the x and y axes, respectively.

Then by the law of parallelogram of vectors,

$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

where $\hat{i}$ and $\hat{j}$ are unit vectors along the x and y axes respectively, and R_x and R_y are the magnitudes of the two components of $\vec{R}$.

Let θ be the angle made by $\vec{R}$ with the x-axis, then

$$\begin{aligned} \cos \theta &= \frac{R_x}{R} \\ \therefore R_x &= R \cos \theta \end{aligned} \quad \text{--- (2.12)}$$

$$\begin{aligned} \sin \theta &= \frac{R_y}{R} \\ \therefore R_y &= R \sin \theta \end{aligned} \quad \text{--- (2.13)}$$

Squaring and adding Eqs. (2.12) and (2.13), we get

$$\begin{aligned} R^2 \cos^2 \theta + R^2 \sin^2 \theta &= R_x^2 + R_y^2 \\ \therefore R^2 &= R_x^2 + R_y^2 \\ \text{or, } R &= \sqrt{R_x^2 + R_y^2} \end{aligned} \quad \text{--- (2.14)}$$

Equation (2.14) gives the magnitude of $\vec{R}$. To find the direction of $\vec{R}$, from Fig. 2.8,

$$\begin{aligned} \tan \theta &= \frac{R_y}{R_x} \\ \therefore \theta &= \tan^{-1} \left(\frac{R_y}{R_x} \right) \end{aligned} \quad \text{--- (2.15)}$$

Similarly, if $\vec{R}_x$, $\vec{R}_y$ and $\vec{R}_z$ are the rectangular components of $\vec{R}$ along the x, y and z axes of the rectangular Cartesian co-ordinate system in three dimensions, then

$$\begin{aligned} \vec{R} &= \vec{R}_x + \vec{R}_y + \vec{R}_z = R_x \hat{i} + R_y \hat{j} + R_z \hat{k} \\ \text{or, } |\vec{R}| &= \sqrt{R_x^2 + R_y^2 + R_z^2} \end{aligned} \quad \text{--- (2.16)}$$

If two vectors are equal, it means that their corresponding components are also equal and vice versa.

$$\text{If } \vec{A} = \vec{B}$$

i.e., if $A_x \hat{i} + A_y \hat{j} + A_z \hat{k} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$, then $A_x = B_x$, $A_y = B_y$ and $A_z = B_z$

Example 2.4: Find a unit vector in the direction of the vector $3\hat{i} + 4\hat{j}$

Solution:

$$\text{Let } \vec{V} = 3\hat{i} + 4\hat{j}$$

$$\text{Magnitude of } \vec{V} = |\vec{V}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$\vec{V} = \hat{\alpha} |\vec{V}|$, where $\hat{\alpha}$ is a unit vector along $\vec{V}$.

$$\hat{\alpha} = \frac{\vec{V}}{|\vec{V}|} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

Example 2.5: Given $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j}$, what are the magnitudes of the two vectors? Are these two vectors equal?

Solution:

$$|\vec{a}| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$|\vec{b}| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

The magnitudes of $\vec{a}$ and $\vec{b}$ are equal. However, their corresponding components are not equal i.e., $a_x \neq b_x$ and $a_y \neq b_y$. Hence, the two vectors are not equal.

2.5 Multiplication of Vectors:

We saw that we can add or subtract vectors of the same type to get resultant vectors of the same type. However, when we multiply vectors of the same or different types, we get a new physical quantity which may either be a scalar (scalar product) or a vector (vector product). Also note that the multiplication of a scalar with a scalar is always a scalar and the multiplication of scalar with a vector is always a vector. Let us now study the characteristics of a scalar product and vector product of two vectors.

2.5.1 Scalar Product (Dot Product):

The scalar product or dot product of two nonzero vectors $\vec{P}$ and $\vec{Q}$ is defined as the product of magnitudes of the two vectors and the cosine of the angle θ between the two vectors.

The scalar product of $\vec{P}$ and $\vec{Q}$ is written as,

$$\vec{P} \cdot \vec{Q} = PQ \cos \theta, \quad \text{--- (2.17)}$$

where θ is the angle between $\vec{P}$ and $\vec{Q}$.

Characteristics of scalar product

(1) The scalar product of two vectors is equivalent to the product of magnitude of one vector with the magnitude of the component of the other vector in the direction of the first.

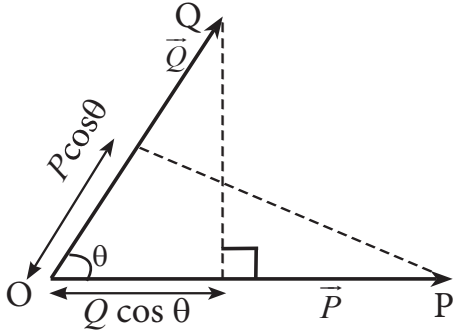


Fig. 2.9: Projection of vectors.

From Fig. 2.9,

$$\begin{aligned} \vec{P} \cdot \vec{Q} &= PQ \cos \theta \\ &= P (Q \cos \theta) \\ &= P (\text{component of } \vec{Q} \text{ in the direction of } \vec{P}) \end{aligned}$$

$$\begin{aligned} \text{Similarly } \vec{P} \cdot \vec{Q} &= Q (P \cos \theta) \\ &= Q (\text{component of } \vec{P} \text{ in the direction of } \vec{Q}) \end{aligned}$$

(2) Scalar product obeys the commutative law of vector multiplication.

$$\vec{P} \cdot \vec{Q} = PQ \cos \theta = QP \cos \theta = \vec{Q} \cdot \vec{P}$$

(3) Scalar product obeys the distributive law of multiplication

$$\vec{P} \cdot (\vec{Q} + \vec{R}) = \vec{P} \cdot \vec{Q} + \vec{P} \cdot \vec{R}$$

(4) Special cases of scalar product $\vec{P} \cdot \vec{Q} = PQ \cos \theta$

(i) If $\theta = 0$, i.e., the two vectors $\vec{P}$ and $\vec{Q}$ are parallel to each other, then

$$\vec{P} \cdot \vec{Q} = PQ \cos \theta = PQ$$

$$\text{Thus, } \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$



Do you know ?

Scalar and vector products are very useful in physics. They make mathematical formulae and their derivation very elegant.

Figure below shows a toy car pulled through a displacement $\vec{S}$. The force $\vec{F}$ responsible for this is not in the direction of $\vec{S}$ but is at an angle θ to it. Component of displacement along the direction of force $\vec{F}$ is $S \cos \theta$. According to the definition, the work done by a force is the product of the force and the displacement in the direction of force. $\therefore W = FS \cos \theta$. According to the definition of scalar product,

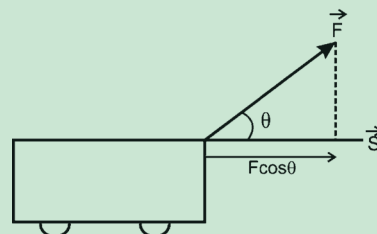
$$\vec{F} \cdot \vec{S} = FS \cos \theta$$

$$\therefore W = \vec{F} \cdot \vec{S}$$

$$\text{Also } W = F (S \cos \theta) = (F \cos \theta) S$$

Hence dot or scalar product is the product of magnitude of one of the vectors and component of the other vector in the direction of the first.

Power is the rate of doing work on a body by an external force $\vec{F}$ assumed to be constant in time. If $\vec{v}$ is the velocity of the body under the action of the force then power P is given by the scalar product of $\vec{F}$ and $\vec{v}$ i.e., $P = \vec{F} \cdot \vec{v}$.



(ii) If $\theta = 180^\circ$, i.e., the two vectors $\vec{P}$ and $\vec{Q}$ are anti-parallel, then

$$\vec{P} \cdot \vec{Q} = PQ \cos 180^\circ = -PQ$$

(iii) If $\theta = 90^\circ$, i.e., the two vectors are perpendicular to each other, then

$$\vec{P} \cdot \vec{Q} = PQ \cos 90^\circ = 0$$

$$\text{Thus, } \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

(5) If $\vec{P} = \vec{Q}$ then $\vec{P} \cdot \vec{Q} = P^2 = Q^2$

(6) Scalar product of vectors expressed in terms of rectangular components :

$$\text{Let } \vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$$

$$\text{and } \vec{Q} = Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k}$$

$$\text{Then } \vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z$$

Proof :

$$\vec{P} \cdot \vec{Q} = (P_x \hat{i} + P_y \hat{j} + P_z \hat{k}) \cdot (Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k})$$

$$= P_x \hat{i} \cdot (Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k})$$

$$+ P_y \hat{j} \cdot (Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k})$$

$$+ P_z \hat{k} \cdot (Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k})$$

$$= (\hat{i} \cdot \hat{i}) P_x Q_x + (\hat{i} \cdot \hat{j}) P_x Q_y + (\hat{i} \cdot \hat{k}) P_x Q_z$$

$$+ (\hat{j} \cdot \hat{i}) P_y Q_x + (\hat{j} \cdot \hat{j}) P_y Q_y + (\hat{j} \cdot \hat{k}) P_y Q_z$$

$$+ (\hat{k} \cdot \hat{i}) P_z Q_x + (\hat{k} \cdot \hat{j}) P_z Q_y + (\hat{k} \cdot \hat{k}) P_z Q_z$$

$$\text{Since, } \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\text{and } \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{i} = \hat{k} \cdot \hat{j} = 0$$

$$\therefore \vec{P} \cdot \vec{Q} = P_x Q_x + 0 + 0$$

$$+ 0 + P_y Q_y + 0$$

$$+ 0 + 0 + P_z Q_z$$

$$\therefore \vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z$$

(7) If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, where $\vec{a} \neq 0$, it is not necessary that $\vec{b} = \vec{c}$. Using the distributive law, we can write $\vec{a} \cdot (\vec{b} - \vec{c}) = 0$. It implies that either $\vec{b} - \vec{c} = 0$ or $\vec{a}$ is perpendicular to $\vec{b} - \vec{c}$. It does not necessarily imply that $\vec{b} - \vec{c} = 0$

Example 2.6: Find the scalar product of the two vectors

$$\vec{v}_1 = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{v}_2 = 3\hat{i} + 4\hat{j} - 5\hat{k}$$

Solution:

$$\begin{aligned} \vec{v}_1 \cdot \vec{v}_2 &= (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 5\hat{k}) \\ &= 1 \times 3 + 2 \times 4 + 3 \times (-5) \\ &= -4 \end{aligned}$$

$$\text{as } \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1,$$

$$\text{and } \hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = \hat{j} \cdot \hat{i} = \hat{k} \cdot \hat{i} = \hat{k} \cdot \hat{j} = 0$$

2.5.2 Vector Product (cross product):

The vector product or cross product of two vectors ($\vec{P}$ and $\vec{Q}$) is a vector whose magnitude is equal to the product of magnitudes of the two vectors and sine of the smaller angle (θ) between the two vectors. The direction of the product vector is given by $\hat{u}_r$ which is a unit vector perpendicular to the plane containing the two vectors and is given by the right hand screw rule. This is shown in Fig. 2.10 (a) and (b)

$$\text{a) } \vec{R} = \vec{P} \times \vec{Q} = PQ \sin \theta \hat{u}_r \quad \text{--- (2.18)}$$

$$\text{b) } \vec{S} = \vec{Q} \times \vec{P} = PQ \sin \theta \hat{u}_s \quad \text{--- (2.19)}$$

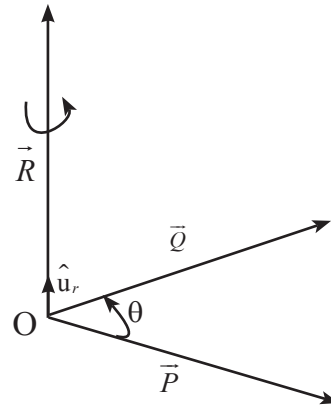


Fig. 2.10 (a): Vector product $\vec{R} = \vec{P} \times \vec{Q}$.

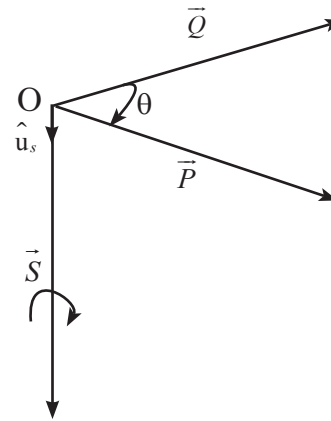


Fig. 2.10 (b): Vector product $\vec{S} = \vec{Q} \times \vec{P}$.